A HIGH SCHOOLER'S GUIDE TO PHYSICS

Learn about the way the world works and simplify your physics learning journey through easy to understand and exhilarating practice problems and content review.

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For our friends, family, and the next generation of physicists and thinkers.

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Introduction

1

Introduction

How to Use This Book

If you are reading this, you are probably trying to learn about physics. Whether you are preparing for a regular high school physics class or even an AP® Physics Exam, this is the perfect book for you. We have crafted a complete guide of every topic in classical mechanics which covers topics in kinematics (the physics of objects in motion) to Simple Harmonic Motion (the physics of oscillation in pendulums and springs). We have practice problems to accompany your learning journey in almost all our chapters and our guides are crafted to help highschoolers, middle schoolers, and anyone understand physics easier. We want to make learning physics easy for you so you can not only get an A in your class but also a 5 on your AP exam or an above average score!

Our book is created by us, Rafi and Zenel. We are highschoolers who have had over 3 years of experience with physics and wanted this book to be the perfect addition to your arsenal to tackle physics. We know physics can be challenging to learn but we are here to make it not only easier for you to learn and understand, but something that is actually fun to learn and pursue in the future. Let's learn and enjoy physics together!

Each chapter rigorously challenges you with unique ways of learning through visual diagrams, fun practice problems, and a perfect content review to teach you in the easiest to understand format.

1.1

Introduction

Significant Figures

What are significant figures? Well, sig figs (short for significant figures) are a measurement of how precise or certain a measurement is. For example, let's look at a glass of water. We can say that the glass contains 1.5 liters of water, but how are we certain that it is exactly 1.5 liters? It is nearly impossible for this glass to contain exactly 1.5 liters, so we take it as an approximation due to the tools available to us, like a measuring cup. This is where sig figs come into play. Essentially, sig figs tell us how certain our measured value is, we can say that the measured amount of water in our cup is 1.5 liters measured to 2 sig figs but if we were to have a more precise measuring cup, we could measure to 3 sig figs, 4 or even more depending on how accurate of a measuring tool we have. Let's say we have a measuring cup that can measure up to 4 significant figures, we could measure to see that the amount of water in our cup is 1.532 liters! Wow, this is a lot more water than we initially thought with only the 2 sig figs from before and this can be crucial when we want to be extremely precise in our calculations. So how do we know how many sig figs we have? There are 6 sig fig rules you must know, which are easy to remember!

Rule #1 - All non-zero digits are significant.

What does this mean? In any number, excluding decimal places, every non-zero digit is considered a significant figure.

Example: 1,234 = 4 sig figs

Example: 2,342,423 = 7 sig figs

Rule #2 - Zeros in the middle of non-zero numbers are significant.

What does this mean? Any number that has zeros in between numbers will have zeros which are significant.

Example: 5.008 = 4 sig figs

Example: 101 = 3 sig figs

Rule #3 - Zeros after the decimal place are significant.

What does this mean? Any zero after the decimal point is considered a significant figure a.k.a trailing zeros are significant AFTER the decimal point.

Example: 43.000 = 5 sig figs

Example: 829.98300 = 8 sig figs

Rule #4 - Zeros and coefficients in scientific notation are significant.

What does this mean? The zeros in scientific notation are significant figures.

Example: $\underline{8.0} \times 10^3$ = 2 sig figs

Example: $\underline{6.022} \times 10^{12}$ = 4 sig figs

Rule #5 - Leading or beginning zeros are NOT significant.

What does this mean? Any zero before other non-zero numbers are not significant, even if they are after or before a decimal point. However, if there is a number before the zeros, then the zeros ARE significant.

Example: $0.00\underline{4}$ - 1 sig fig

Example: $\underline{1000.04}$ - 6 sig figs

Rule #6 - Zeros in a large number without a decimal are NOT significant.

What does this mean? If there is a large whole number (a number without decimal places) that has zeros at the end with no other whole number, then the zeros are not sig figs.

Example: $\underline{4}000$ = 1 sig fig

Example: $\underline{52{,}342}{,}000$ = 5 sig figs

Try it on your own:

How many sig figs do these numbers have:

1. 4.0283000 = __________________ sig figs

2. 98.0002 = __________________ sig figs

3. 291392 = __________________ sig figs

4. 1,952 x 1013 = __________________ sig figs

5. 0.002 = __________________ sig figs

6. 102,000 = __________________ sig figs

Answers:

1. 8 sig figs - Rule #3 - Zeros after the decimal are significant
2. 6 sig figs - Rule #2 - Zeros in the middle of non-zero numbers are significant
3. 6 sig figs - Rule #1 - All numbers that are not zero are significant
4. 4 sig figs - Rule #4 - Zeros and coefficients in scientific notation are significant
5. 1 sig fig - Rule #5 - Leading or beginning zeros are NOT significant
6. 3 sig figs - Rule #6 - Zeros in a large number without a decimal are NOT significant.

Well now we know about the basic rules of significant figures, let's learn about addition, subtraction, multiplication and division using sig figs, it’s very simple!

Adding & Subtracting - Look for the number with the LEAST number of decimal places.

What does this mean? Look for the number with the least number of decimal places and this will be the number of decimals we will round to.

Example: 2.34 + 5004 + 481.44 = 5487.78

⬆ Zero decimal places, round number to 0 decimal places

5487.78 ≈ 5488 (round up)

Example: 5.82 + 98.213 + 43.1 = 147.133

⬆ 1 decimal place, round number to 1 decimal place

147.133 ≈ 147.1 (round down)

Multiplying & Dividing - Look for the LEAST amount of sig figs.

What does this mean? We will round to the amount of sig figs as the factor with the least amount of sig figs.

Example: 24.5 × 63.2751 = 1550.23995

⬆ 3 sig figs ⬆7 sig figs = the least amount of sig figs is 3, we will round the answer to 3 sig figs

<u>1550.23995</u> = 9 sig figs = round to 3 sig figs (starting from the leftmost digit)

<u>155</u>0 = 3 sig figs

Example: 102.02 / 72.4 = 1.4091160221

⬆ 5 sig figs ⬆3 sig figs = the least amount of sig figs is 3, we will round the answer to 3 sig figs

1.4091160221 = 11 sig figs = round to 3 sig figs (starting from the leftmost digit)

1.41 = 3 sig fig

Try it on your own:

Round your answers to each equation to the correct number of sig figs (calculator allowed):

1. 382.87 + 782.356 + 23.4 + 76,292.9887 ≈

   ____

2. 87.34 × 928.22 × 987.233 ≈

   ____

Challenge Question:

452.64 × 2342.23 ÷ 456.23 + 623.52 + 82.4 ≈

____

Answers:

77481.6 - The lowest number of decimal places in any of the addends is just 1, so round the answer (77481.6147) to 1 decimal place

80040000 - The lowest number of sig figs in any of the factors is 4 (87.34), so round the answer (80035704.7288) to 4 sig figs

3029.72 - Split this question into 3 parts, make sure to use P.E.M.D.A.S (or any other acronym for the order of operations) first multiply 452.64 and 2342.23, round to the correct number of sig figs (5 sig figs) so you will get 1,060,200. Next divide 1,060,200 by 456.23, again round to the correct number of sig figs (5 sig figs). You will get 2323.8. Now for the final step, add 2323.8 to 623.52 and 82.4 using the addition rule for sig figs. You will round to 1 decimal place and your final answer will be 3029.72. Our answer without using sig fig rules would be 3029.71937137 which is close to our answer using the sig fig rules.

Now you are equipped to tackle any physics, mathematics, chemistry, or other class that involves significant figures. However, you likely won't need to use significant figures extensively in high school physics classes, as the process can be tedious. It's generally sufficient to round your answers to 2 or 3 decimal places when you encounter decimals.

1.2

Introduction

Significant Units

In order to understand our universe, we need a way to measure physical quantities, like mass, time and distance. Without a standard value to quantify measurements, it becomes nearly impossible to give meaning to physical quantities – imagine having to tell someone how long something takes without using standard values of time, such as seconds or minutes. It's like your friend saying, "I will be at your house in 15 bananas." To your friend bananas may be equal to 15 minutes but to you, you have no idea! With this in mind, scientists sought out to create standard, physical constants, which we now call units.

These units are standard, people from Germany to Mozambique and across the world use the same units. This ensures that there is no confusion between ways of measuring values and these values will not change throughout time. These units were measured differently before, for example a second was equal to 1/86,400 of a day, but as the Earth's rotation slows down, this value is changing, as a result scientists changed what exactly a second is equal to and now the value is standardized to something constant.

All measured quantities can be expressed by fundamental units, units which are not dependent on any other. These fundamental units are the meter, the second, the kilogram, the ampere, the mole, the kelvin and the candela. In this book, we will only observe the meter, the second, and the kilogram but the other units are still very important in higher level physics courses.

The meter (m) is the unit representing distance. Using the meter, we can quantify the length between two points. For example, the length of a football field is 109.7 meters, which is the distance between one side of the field to another. The meter is defined as the distance light travels within 1/299,792,458th of a second. The second (s) is the unit representing time. With the second, we can observe time which has passed. Seconds are defined to be the time it takes for Cesium to vibrate 9,192,631,770 times. The kilogram (kg) is the unit representing mass. Mass is defined to be a measure of inertia, which will be further explained in Chapter 3, but for now think of it as how heavy something is. The kilogram is defined in terms of the Planck constant, the speed of light and the second. The Planck constant is used in quantum mechanics. These are all ideas we will be discussing throughout our book, and we will learn about other units along the way!

Scientists often must deal with extremely large or extremely miniscule quantities, to the point where it is inconvenient to represent these quantities using these base units – which is why we have developed 'metric prefixes.' These prefixes are in factors of ten, like how a gram is 10^{-3} of a kilogram, or how a kilometer is 10^{3} of a meter. This allows us to conveniently describe these physical quantities and use them in calculations. Below we have included a convenient table to help you convert between some of the common prefixes.

Prefix:	Magnitude:	Symbol:
peta	10^{15}	P
tera	10^{12}	T
giga	10^{9}	G
mega	10^{6}	M
kilo	10^{3}	k
hecto	10^{2}	h
deka	10^{1}	da
(N/A)	1	(N/A)
deci	10^{-1}	d
centi	10^{-2}	c
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p
femto	10^{-15}	f

People often must convert units across these magnitudes, which is why they have developed an intuitive and easy way to do so – dimensional analysis.

Dimensional analysis begins by finding the units that you want to convert, so for example, let's use the gram and the kilogram. Now, we need to find the ratio between these units, or in other words, how many grams are in a kilogram. Using the table, we can see that there are 1000 grams in 1 kilogram. Finally, we can set up our final dimensional analysis equation. For this, we will use 50 grams as an example, but any value will work.

$$50\ g \times \frac{1\ kg}{1000\ g} = 0.05\ kg$$

Let's examine what happened a little closer. We take our initial value with its units, grams, and multiply it with the ratio between the two units, which are kilograms and grams. The grams cancel each other out, leaving us with just the kilograms and a simple mathematical equation: $\frac{50}{1000}$. This gives us our final answer of 0.05 kilograms.

Try it on your own:

Use the chart to convert the following units:

1. 46 kilometers = ____________ meters
2. 923 nanoseconds = ___________ microseconds
3. 67 hectograms = ____________ megagrams
4. 58924712 picometers = __________ dekameters

Answers:

46000 meters

0.923 microseconds

0.0067 megagrams

0.0000058924712 dekameters

1.3

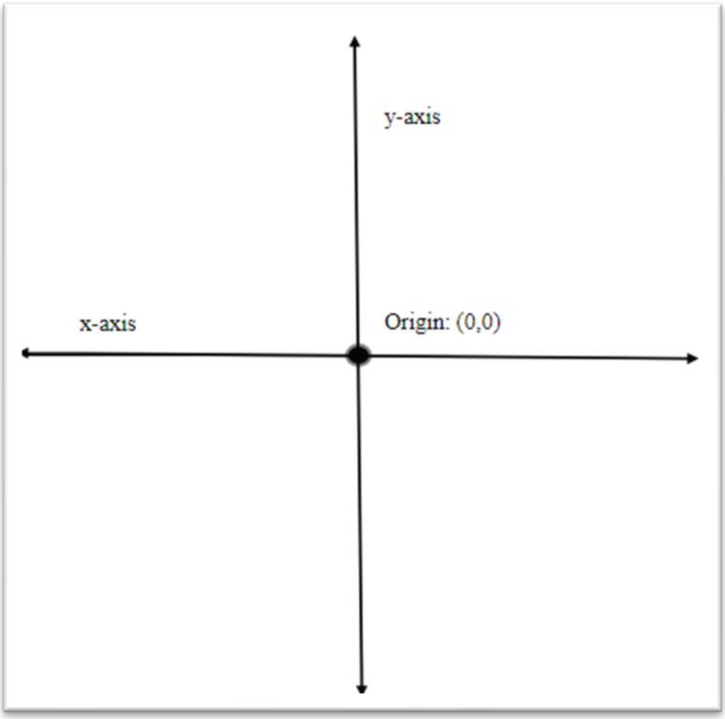
Introduction

Coordinate System

Graphs are a convenient way to represent relationships between quantities. But in order to use graphs, we need to set one up first! You might be wondering; how do we do that? Well, it all starts with one point. We will name this point the origin, and everything that will be represented in this graph will be in respect to it. The coordinates of the origin are (0,0).

Coming out of the origin point are axes. An axis is a number line which goes on forever in both directions. Typically, there are two axes: the x-axis, or horizontal axis, and the y-axis, or the vertical axis.

What does each of these represent? The x-axis represents something called an independent variable. The value of an independent variable is not affected by the change of another variable. For example, the amount of light a plant receives. The y-axis represents a dependent variable. Dependent variables are dependent on the independent variable, their value is determined by the value of the independent variable. Going off the previous example, the dependent variable would be the height of the plant. In physics, we often observe how quantities change over time, so it is extremely common for the x-axis to be time and the y-axis to be another value dependent on time such as distance traveled, temperature and speed.



Finally, we need to determine which 'direction' of the axes are positive and negative. In math class, we consider positive to be upwards or rightwards depending on the axis, and negative to be downwards or leftwards, again, depending on the axis. However, in physics, it may be more convenient to play around with these directions, like making downwards on the y-axis positive and upwards negative, like when you're dealing with falling objects. Since they're only moving downwards, it's more convenient to represent those values as positive than strictly negative values. This is not necessary, but it can definitely be of use!

Although creating a coordinate system is common knowledge, it is important to lay a solid foundation and understanding, that way we can avoid confusion in the future.

1.4

Introduction

Vectors & Scalars

Vectors and Scalars are something you will constantly be hearing throughout this book, but what are they? Vectors and scalars are two different ways to represent quantities. For example, you may look at the speedometer of your car on the highway and it says “60 mph.” This is a quantity but is it a scalar or a vector? Well, here is the difference:

Scalars - Quantities that ONLY have Magnitude (an amount of something)

Vectors - Quantities that have BOTH Magnitude and Direction

For our example above, 60 mph (miles per hour) is a quantity with a magnitude, or an amount of something, in our case speed. There is no mention of which direction the car is traveling so it is not a vector. So how can we represent our car’s speed with a direction a.k.a turning our scalar into a vector? It’s simple! Add a direction into the value, instead of just 60 mph, we can say 60 mph north, east, southeast, etc. Now we have a magnitude and direction, our quantity is now a vector. This is especially important when we talk about specific variables in physics. Take for instance speed and velocity, speed is the scalar version of velocity. Speed has no direction whereas velocity does. That's why you may see the symbol for velocity (v) represented with an arrow pointing to the right ($\vec{v}$) which means it is a vector quantity known as velocity. Speed is usually just represented as (v) since without the vector symbol it is not a vector quantity.

Here are some examples of scalar quantities:

Mass, speed, distance, time, energy, density, volume, temperature, distance, work

Here are some examples of vector quantities:

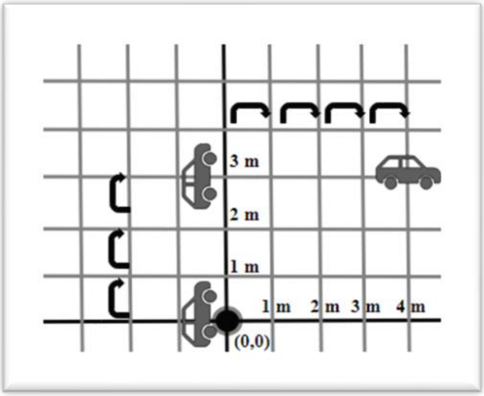
Force, displacement, velocity, acceleration, momentum, friction, weight

Now that we know what vectors and scalars are, let's talk about one of the most important differences between a scalar and vector quantity, distance vs displacement. What is distance? Distance is the total length an object has moved from its starting position to its final position. We represent distance using (d). Since there is no vector symbol, we know that this is a scalar value, what does this mean in this context? It's a scalar, distance has no direction, so you won't say "15 meters east" but rather just "15 meters." Remember it's the TOTAL change in position of an object from its starting position. Now you may be wondering, what is displacement? They sound similar and honestly have a very similar meaning, but they are two completely different ideas. Displacement is the value of how much an object has moved as compared to its starting position. It is **NOT THE TOTAL** amount it is moved but rather the shortest distance from its starting position to its final position. We represent displacement using ($\Delta\vec{x}$) and since you see the vector symbol, we know it is a vector quantity which has a direction. We use the Greek letter "Δ" (delta), which means change in, to help us represent this value. For this we will not say

"15 meters" like we did for representing distance, we will instead say "15 meters northeast." This still may be confusing so here's another example:

A car travels 3 meters north then it takes a sharp right turn and travels 4 meters east. Find the car's distance traveled and displacement.

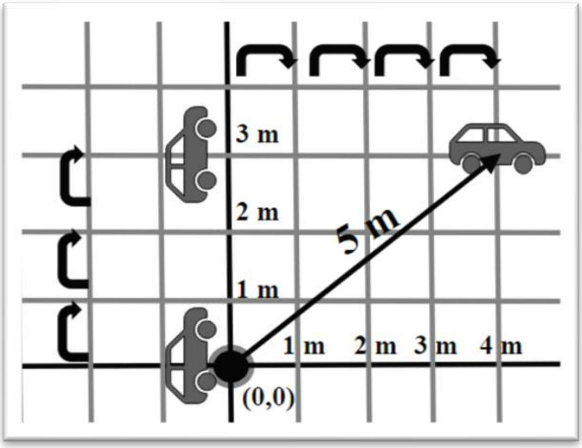
Here we can draw a diagram to help us. Remember to use your rules of the coordinate system!



To find the total distance the car has traveled is very simple, we just add up how far the car traveled.

3 meters north + 4 meters east = 7 meters. The distance traveled = 7 meters

Finding the displacement is a little tricky, remember, displacement is the shortest path from the starting position to the final position. Does the movement of the car seem familiar? Yes, it should, it resembles a triangle! If we remember from geometry class, the shortest path between the two points, the origin and the end position, is the hypotenuse. The equation to find the length of the hypotenuse is Pythagorean’s Theorem which is ($a^2+b^2=c^2$) where a and b represent one leg of the triangle and c represents the hypotenuse. To solve for the hypotenuse, or the displacement, let’s plug in the values of a and b. (32 + 42 = c^2) This equals to (9 + 16 = c^2) which then equals to c^2 = 25. Square rooting on both sides gives us our final answer as 5. This means that the displacement of the car was 5 meters, but don't forget, displacement is a vector quantity which means we need a direction! Since the car traveled North and then East, with respect to the starting position, we can say that the final displacement is 5 meters North-East.



Try it on your own:

1. A man is running in a track meet, he must run the track for 4 full laps and return to his starting position. The length of one lap is 400 meters. What is the man's distance traveled and displacement? Answer in terms of kilometers.

Distance: ________________ meters

Displacement: ________________ meters

2. A rocket ship is launching towards Mars, it begins on Earth and travels to Mars, the captain of the ship wants to go farther, he turns the thrusters on and goes towards Jupiter but stops halfway. The captain directs the ship back to Mars. What is the distance and displacement traveled in meters? Assume that the distance between Earth and Mars is 3.6×10^{11} meters, the distance between Mars and Jupiter is 5.5×10^{14} meters and assume that the planets do not orbit and are in a straight line starting with Earth then Mars and then Jupiter from left to right.

Distance: ________________ meters
Displacement: ________________ meters

3. Sir Isaac Newton wants to fish for a big fish. He decides to cast his reel 5 meters north of the pier, he has no luck and decides to cast it 3 meters east of his previous spot, again he has no luck. He casts his reel 10 meters north from the pier where he finally catches his dinner for the evening! How far did the fishing line travel throughout his struggles in terms of distance and displacement, answer in meters. Assume he had to reel the fishing line back to the pier every time.

Distance: _________________ meters
Displacement: _________________ meters

Answer Key:

1. Distance: 1.6 km

Displacement: 0 km

Explanation: The man must run the track for 4 laps, each lap is equal to 400 meters. To find the total distance traveled we multiply 4 by 400 to get 1,600 meters. Now we must do unit conversions, 1 km is equal to 1,000 meters. We can divide 1,600 meters by 1,000 meters to get our answer of 1.6 km. Now to find the displacement, since the starting position and the final position are the exact same, the person hasn't moved at all from his starting position! This means that the person hasn't been displaced at all and we get 0 km of displacement.

2. Distance: 5.5036×10^{14} meters

Displacement: 3.6×10^{11} meters to the right (or east)

Explanation: Let's split this question up into parts, let's try to comprehend what it is saying. The captain first travels to Mars from Earth, which is 3.6×10^{11} meters. Then, he travels half-way to Jupiter, the distance between Mars and Jupiter is 5.5×10^{14} meters. To find half the distance we can divide 5.5×10^{14} by 2. We get 2.75×10^{14}. Then, the captain turns around and returns to Mars. This is the same distance he just traveled which is 2.75×10^{14}. To find the total distance we can add all these values together. For all the exponents to be equal to 14 we must convert 3.6×10^{11} to a value x 10^{11} to do so we can move the decimal place of 3.6 until its value can be multiplied by 110^{14}. We now get

0.0036×10^{14}. Now that all the exponents are equal, we can add the values.

$0.0036 \times 10^{14} + 2.75 \times 10^{14} + 2.75 \times 10^{14} = 5.5036 \times 10^{14}$.

The total distance traveled by the rocket ship is

5.5036 x 1014 meters.

To find the displacement we take the shortest distance between the origin and the final position. The final position is just Mars, since the distance between Earth and Mars is 3.6×10^{11} meters our displacement is 3.6×10^{11} meters. However, we also need direction. Since the planets are ascending from left to right (Earth, Mars, Jupiter) we can say that the direction is right or east, and our final displacement is 3.6×10^{11} meters to the right or east.

3. Distance: 46 meters

Displacement: 0 meters

Explanation: To find the total distance the line has traveled let's look at the question piece by piece. First it is cast 5 meters, then reeled back 5 more meters, then cast 5 meters again and 3 meters, it returns that same distance, so 8 more meters, and finally it is cast 10 meters away and returns 10 meters. Let's add up all the distances for our total distance.

$5 + 5 + 5 + 3 + 8 + 10 + 10 = 46$ meters of distance. For the displacement the final position is the pier which is our origin which means there is no displacement.

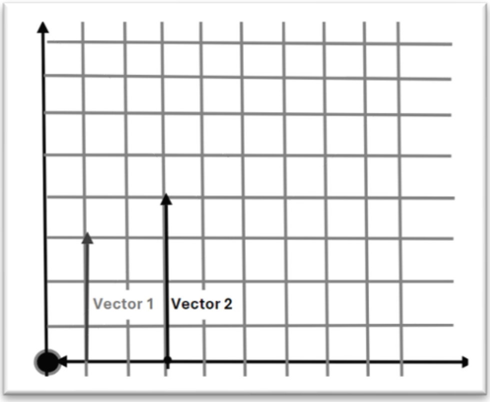
1.5

Introduction

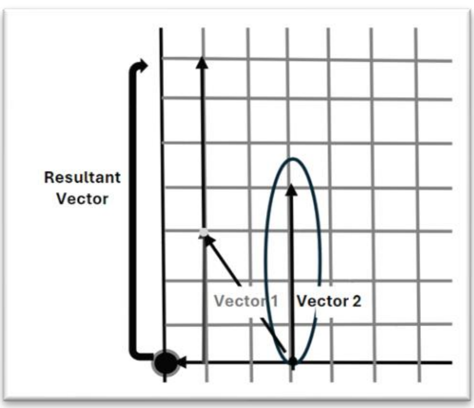
Vector Addition

We have talked about vectors, and we learned how to use them to determine the distance or displacement of any object but how can we add vectors? Well, we already started doing this when we used the Pythagorean Theorem to find the length of the hypotenuse. We use something called the "Head to Tail" method to add vectors and to find the "resultant vector." This resultant vector is essentially the displacement of an object, and we already experienced an example of this in the vectors & scalars unit. To answer this question, we need to use the rules of vector addition and implement the "Head to Tail" method. So, what is the Head to Tail method? The Head to Tail method states that to add two vectors together we place the tail (or the non-arrowhead) of the 2nd vector to the "head" (the arrowhead) of the first vector.

Let's draw a diagram to wrap our heads around this concept.

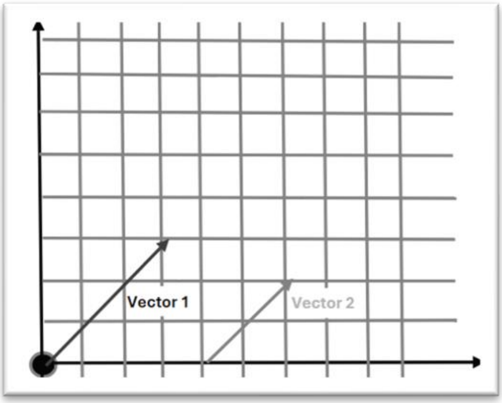


Here you can see two vectors, vector 1 and vector 2. To add these two vectors together, we take the tail (the side of the vector without an arrowhead) of vector 2, add place it on top of the head (the side of the vector with an arrowhead) to find our resultant vector.

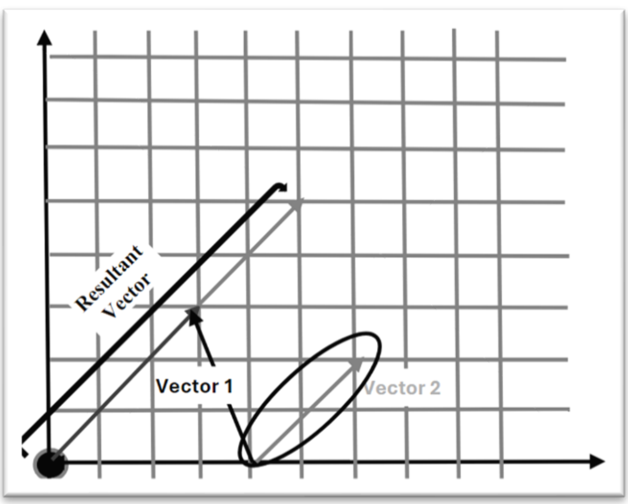


As you can see, we moved the tail of vector 2 to the head of vector 1 and we discovered our resultant vector.

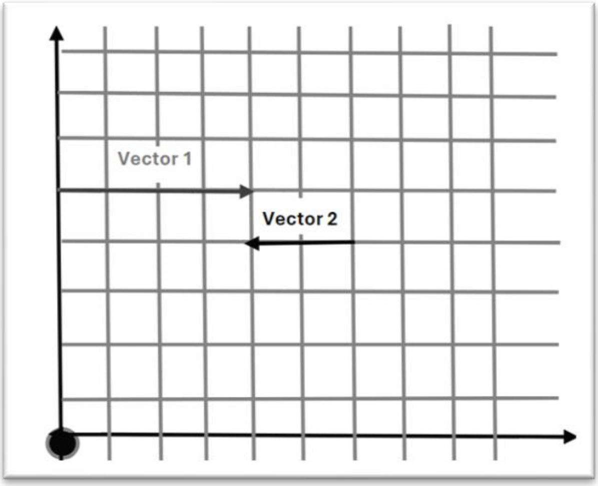
The head to tail method isn't only for horizontal or vertical vectors, we can also use it for vectors placed at an angle as shown below.



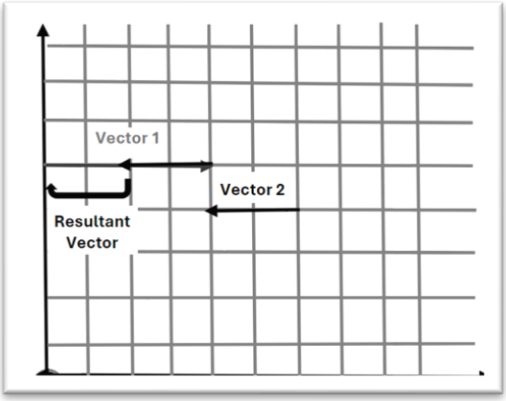
Here we have two vectors placed at an angle; to find the resultant vector we use the same method as before; we place the tail on the head of vector 1.



Now that we know about vector addition, let's move on to vector subtraction! Let's think of subtraction in a new way, you aren't "removing" a value but rather you are adding the negative of that number. For example, we know that 5 minus 2 is equal to 3, but we can also think of it as 5 plus negative 2 is equal to 3. This is essentially what we are doing when we are doing vector subtraction. Instead of "subtracting a vector" we are "adding its negative" by negative we do not mean the negative value of the vector but rather the negative or "opposite direction" of the vector. Let's draw a diagram to better help us understand this idea:



Here we have two vectors, one pointing right (vector 1) and one pointing left (vector 2). To subtract these two vectors (or technically speaking add them together) we use the Head to Tail method!



The resultant vector is smaller than vector 1 and it will be pointing to the right side.

Chapter 2

Kinematics

2.1

Kinematics

Position and Velocity Over Time

Kinematics is the study of the motion of objects. These objects may stay stationary, speed up, slow down, or move at a constant rate. One of the most integral parts of kinematics is observing an object's position over time – so how do we do that?

When we talk about position, we mean the shortest distance between two points: the origin and the object

Does this sound familiar? It is the same as displacement. Position and displacement are two words for the same idea!

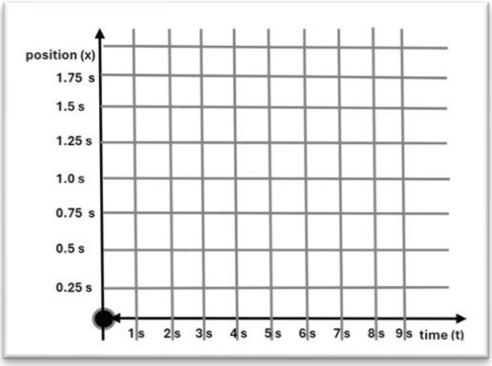
For example, if your house is the origin and your school is 1000 meters away from your house, then your school's position is 1000 meters. But 1000 meters where? East? West? This is why position is a vector, it has a direction. When we talk of an object's position, we also speak of the direction of it relative to our reference point, or the origin.

Now that we understand what position is, take a look at any still object for a couple seconds. Its position did not move for the entirety of the time we were observing it. So how can we represent this information on a graph?

Let's set up our graph first. We need to define our two axes. The variables we are comparing are 'time' and 'position'. Since the passage of time does not depend on the position of the object, we will choose it to be represented by the x-axis and position to be represented by the y-axis. We will also need a scale for our graph, so let's make each interval on the x-axis worth one second and each interval on the y-axis worth 0.25 meters.

We said that the position of the object is the same throughout, but what is it? That all depends on the origin that you chose. The object could be its own origin point, but for simplicity's sake let's choose a nearby object that is 0.5 meters to the right, meaning that relative to that object, the object we are looking at has a position of 0.5 meters horizontally.

Now that we know the value for position and have our graph set up, we can represent the position of the object over time. Plot the object's position for each second that passes. Eventually, when you have done this for a couple of seconds, you will find that the points all have the same y-coordinate



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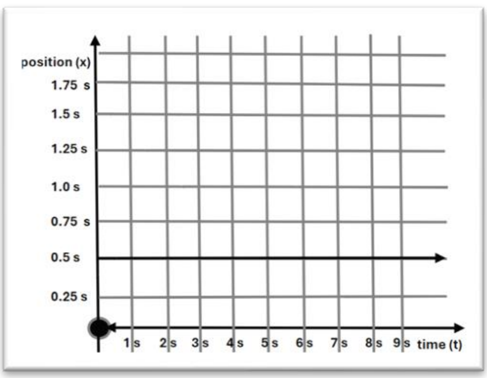
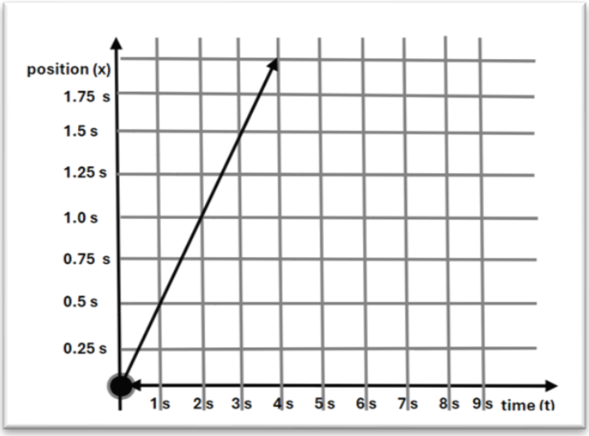
Connecting all the points gives us a straight line at y = 0.5, meaning that the object's position will always be 0.5 despite the passage of time.



What about an object whose position changes over time?

Let's look at an object whose position changes by 0.5 meters every second. If we plot a point for this object's position every second, it should look something like this:



As you can see, we have a linear line with a constant slope. The slope of this line is incredibly important, as it is the velocity of the object. Velocity is a measure of how much an object is displaced every second and it is a vector quantity. We can find the velocity by measuring the change of position and dividing it by the change of time. Since the position changes 0.5 meters each second, then our velocity is 0.5 m/s.

We can also take velocity and graph it over time as well! But why should we? Well, doing so allows us to investigate the velocity, and tells us about the displacement of the object and the acceleration... which we'll speak more about in the next section.

Since the slope of the line is constant in the first graph, then this means that the velocity is constant as well. But what does this graph tell us about the displacement and the acceleration of the object? Let's take a closer look. If we take the area under the velocity line, this gives us the total displacement of the object. But why is that? We know that velocity is the change of position divided by the change of time, and if we multiply the change of time with this, we end up with just the change of position – displacement! This ends up being a useful tool for us if we want to find the displacement but are just given the velocity.

Well now you may wonder, what is acceleration? Let's find out in the next chapter!

2.2

Kinematics

Acceleration

Vroom vroom... When we are in a car and the driver presses the gas pedal the car moves faster and faster. Its velocity increases, and as a result, there is a change in the velocity over a certain period of time. This change in velocity over time is called acceleration. Acceleration describes how much an object speeds up over a period or the rate of change of its velocity. We write acceleration with the symbol “a: and its equation is $a = \frac{\Delta v}{\Delta t}$. As you can see acceleration is equal to the change in velocity over the change in time or it can also be written as $a = \frac{(v_f - v_i)}{\Delta t}$. Essentially all we do to find the acceleration is take the final velocity and subtract it from the initial velocity and then divide it by the time it takes to reach the final velocity. The units are m/s^2 also known as meters per second per second. We can better understand acceleration by seeing an example of it. Let's take a look:

A car travels at a constant velocity on the freeway until the cops start to approach the driver, the driver hits the gas and speeds up from 5 m/s to 15 m/s in 10 seconds! What was the driver’s acceleration?

To answer this question let's use our new equation. The final velocity of the car was 15 m/s, and its initial velocity was 5 m/s. So v_f = 15 m/s and v_i = 5m/s. The car speeds up in a period of 10 seconds so Δt = 10 s. When we solve the equation $a = \frac{(15m/s - 5\,m/s)}{10\,s}$, we get an answer of 1 m/s² which means the car's velocity increases by 1 m/s every second.

Acceleration is very important as it is a way to model the change in an object's velocity. Since velocity is a vector, it has a magnitude and direction which means if acceleration changes either its direction or magnitude can change. If an object is moving 10 m/s east but then it starts moving 10 m/s west in 2 seconds, there is a change in the acceleration.

Let's solve a problem to help us understand. When we graph such a scenario, we usually label east to be the positive x-axis (+x) direction and west to be the negative x-axis (-x) direction. So, we can write v_f as -10 m/s and v_i as 10 m/s. Once we plug it into our equation, we get $a = \frac{(-10\,m/s - 10\,m/s)}{2\,s}$, our answer is -10 m/s². If we didn't consider direction, we would get a totally different answer so make sure you always consider direction when you work with acceleration! We will understand later with circular motion why the direction being considered for acceleration is so significant.

Try it on your own:

1. Alfred likes to run so he begins his day with a simple jog going at 2 m/s for 10 seconds. He suddenly begins to speed up to a speed of 5 m/s in just 8 seconds. What was his acceleration during the period he sped up?

   Acceleration: ________________ m/s^2

2. John is in his new Lamborghini Huracán cruising down the freeway at 25 m/s. He suddenly sees a police officer tailing him, so he begins to press on the gas pedal accelerating to a speed of 67 m/s! The Lamborghini is very powerful and speeds up in just 7 seconds! What was the acceleration of John and his Lamborghini?

   Acceleration: ________________ m/s^2

3. A runner is running a 1k race, he begins at a pace of 5 m/s and slows down to 3 m/s in 2 seconds. He continues running at this speed for 2 minutes until it's the last 100 meters of the race where he speeds up to 15 m/s in 4 seconds. What was the runner's acceleration during the 3 periods of time when he is running?

Acceleration from 5 m/s to 3 m/s: Acceleration:

_________________ m/s^2

Acceleration during 2-minute interval: Acceleration:

_________________ m/s^2

Acceleration from 3 m/s to 15 m/s: Acceleration:

_________________ m/s^2

Answer:

1. To solve this problem, we can simply just plug the values into our equation for acceleration. Be careful! Since it says that Alfred speeds up in 8 seconds the change in time for the equation is 8 seconds NOT 10 seconds as there was no change in velocity during those 10 seconds. $a = \frac{(5\ m/s - 2\ m/s)}{8\ s}$. We get our final acceleration as 0.375 m/s^2.

2. For this question, we can do the same thing again, let's plug the values into our equation $a = \frac{(67\ m/s - 25\ m/s)}{7\ s}$. We get about 5.71 m/s^2!

3. This question is asking for 3 different answers so let's break it down into 3 parts. The acceleration between the change in velocity from 5 m/s to 3 m/s is calculated by plugging the values into the equation $a = \frac{(3\ m/s - 5\ m/s)}{2\ s}$, to get a value of -1 m/s^2 which is a negative acceleration (also known as a deceleration). During the 2-minute interval, we first convert the minutes into seconds. 2 minutes x 60 seconds = 120 seconds. The final velocity and initial velocity are the same so the change in velocity is 0! The time interval doesn't matter as 0 divided by anything is just 0 so the acceleration during that period is 0 m/s^2. To calculate the acceleration between 3 m/s to 15 m/s we will just plug the values into our equation to get $a = \frac{(15\ m/s - 3\ m/s)}{4\ s}$. The acceleration during that period is 3 m/s^2.

2.3

Kinematics

Kinematic Equations

When we talk about kinematics, we talk about representing an object's motion using math and geometry. We have already discussed various ways to represent the motion of an object but how exactly can we solve for some specific things that our other equations can't provide us? Here's where the kinematic equations come in. There are three main equations that we can manipulate to find just about everything we need to depict an object's motion; this includes its change in position (displacement), initial velocity, final velocity, time, and acceleration. The three main equations are below and solve for three different components:

1. $\Delta x = v_i t + \left(\frac{1}{2}\right) a t^2$
2. $V_f^2 = v_i^2 + 2a\Delta x$
3. $V_f = v_j + at$

There are many ways to write these various equations, but we will be using this format throughout this book. The first equation solves for the change in position of an object also known as displacement. The second equation solves for the final velocity of an object when time is unknown, and the third equation solves for the final velocity of an object when time is known. These equations can be manipulated to solve for other parts of the equation such as finding time when final velocity, initial velocity, and acceleration are known when using equation three. Let's see how to use these equations!

Steps for solving questions about kinematics:

1. List out the knowns and the unknowns
2. Select which equation(s) to use
3. Plug in values to solve for unknown values
4. Let's put these steps into use.

Let's put these steps into use.

Henry begins his daily run jogging at a speed of 1 m/s but then suddenly starts to accelerate at 3 m/s^2 for 3 seconds. How far did Henry travel during his run?

To solve this let's follow our steps, what are the knowns and the unknowns? We know that he initially started his run at 1 m/s which is v_i, and we also know that he accelerated at 3 m/s^2 which means a = 3 m/s^2, and finally, the time for his acceleration was 3 seconds so t = 3 s. We are looking for Δx which is what the equation is asking for. The equation which has all these parts is equation number one! Let's plug in our values and solve:

$$\Delta x = v_i t + \left(\frac{1}{2}\right) a t^2$$

$\Delta x = (1\ m/s)(3\ s) + \left(\frac{1}{2}\right)(3\ m/s^2)(3\ s^2)$ and we get $\Delta x = 7.5$ meters as Henry's displacement.

Let's try a harder problem that requires the usage of more than one of these equations:

Bob likes to ride on roller coasters. Today he rode a roller coaster that was initially at rest but began to accelerate very fast at 10 m/s² and was displaced 200 meters during this time. How long did Bob ride the rollercoaster?

Let's use our steps and list out the knowns and the unknowns: Initial velocity is equal to 0 m/s as it begins at rest, acceleration is 10 m/s², and displacement is 200 meters. The unknown we are looking for is time, none of the equations we have can solve for time from just these three knowns so we must do something else first. If we use the equation, $V_f^2 = v_i^2 + 2a\Delta x$ we can solve for final velocity and then plug in that value into either equation one or equation three to find time! Let's try it out, first plug in the values into equation two.
$V_f^2 = (\ 0\ m/s) + 2(10\ m/s^2)(200\ m)$ if we solve for V_f we get 200 m/s. Now let's plug in this value along with our other knowns into the equation $V_f = v_j + at$
$(200\ m/s) = (\ 0\ m/s) + (10\ m/s^2)t$. Once we isolate t and solve for the time, we get t = 2 seconds!

Try it on your own:

1. A football player throws a ball with an initial velocity of 15 m/s to a friend. The friend catches the ball in 3 seconds. How far away was the friend?

Distance: ________________ meters

2. Sarah runs a race in just 8 seconds. She began her run initially starting at rest but accelerated rapidly at 3 m/s^2. How fast does she run after 20 seconds?

Final Speed: ________________ m/s

3. Marco is in space riding his spaceship traveling at a speed of 1,000 m/s initially, he decelerates before he hits an asteroid that is 35 meters away from him. What must be Marco's acceleration so that he does not hit the asteroid?

Acceleration: ________________ m/s^2

Answer Key:

1. Let's list out our knowns, initial velocity is 15 m/s, and the time is 3 seconds. The unknown is distance or change in position so we will use equation one, $\Delta x = v_i t + \left(\frac{1}{2}\right) a t^2$. Since there is no acceleration on the ball, we will assume it is 0. When we plug in the values, we get $\Delta x = (15\ m/s)(3\ s) + \left(\frac{1}{2}\right)(0\ )(3s)^2$ where the change in position or distance is equal to 45 meters.

2. To start let's list out our known values, the time is 8 seconds, the initial velocity is 0 m/s as she starts at rest, and finally, her acceleration is 3 m/s^2. We are solving for final velocity so we will use equation three, $V_f = v_j + at$. Let's plug in the known values to solve for the final velocity. $V_f = (0\ m/s) + (3\ m/s^2)(8\ s)$ and we get a final velocity of 24 m/s.

3. To answer this question let's look at the knowns, initial velocity is 1,000 m/s, distance is 35 meters, final velocity is unknown and so is acceleration which is what we are solving for. But wait! Final velocity is actually known, the spaceship must stop completely to not hit the asteroid which means that final velocity must be equal to 0 m/s! Since we have all this information, we can find acceleration using equation two, $V_f^2 = v_i^2 + 2a\Delta x$. Plugging in the known values we get $(0\ m/s)^2 = (1000\ m/s)^2 + 2a(35\ m)$ if we solve for acceleration, we get an acceleration of -14,285.71 m/s^2!

2.4

Kinematics

Freefall & Gravity

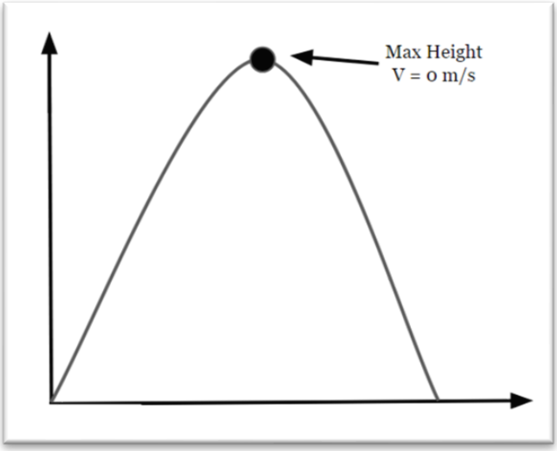
Ouch! An apple just fell on my head. Hmmm... If an apple falls, then why doesn't the moon fall? This is probably the most famous story regarding the concept of gravity and I'm sure we have all heard it before. Gravity was first discovered by Isaac Newton, and it is a force (we will talk more about forces in the next chapter) that makes bodies attract and move towards each other. Well, when we talk about gravity in kinematics, we talk about the acceleration due to gravity which we write as “g” (also known as ‘little g’). The acceleration due to gravity has a value that equals about 9.81 m/s^2 but we will just be using -10 m/s^2 to make calculations easier. When an object is in freefall it means that the only acceleration on the object is from gravity.

Let's look at an example:

Jack throws a ball into the air with an initial velocity of 12 m/s. The ball reaches its max height and comes back down, the ball experiences freefall throughout the fall. How high did it reach?

Height: ________________ meters

To solve this question, we must keep one thing in mind. At the top (max height) of an object being thrown into the air, the velocity of the object is equal to 0 m/s.



Here the graph just depicts what the trajectory of an object thrown into the air would look like, what the axis represents does not matter.

This is very useful for us as we can split up the motion of the object into two parts, one where it is reaching its max height and the other part where it is falling back down. Now we can list our knowns and unknowns, we know that the initial velocity is 12 m/s, and we also know that the velocity at the max height is equal to 0 m/s so the final velocity at the top is equal to 0 m/s, and the acceleration is equal to g which is -10 m/s^2 and we are solving for max height which is the change in position in the y-axis. We can use equation two $V_f^2 = v_i^2 + 2a\Delta y$ to solve our question, here Δx is represented as Δy as the motion is in the y-direction (we can't use equation one as we don't have time). Let's plug in our knowns to solve for max height: $(0\ m/s)^2 = (12\ m/s)^2 + 2(-10\ m/s^2)\Delta y$, if we isolate for Δy we get 7.2 meters.

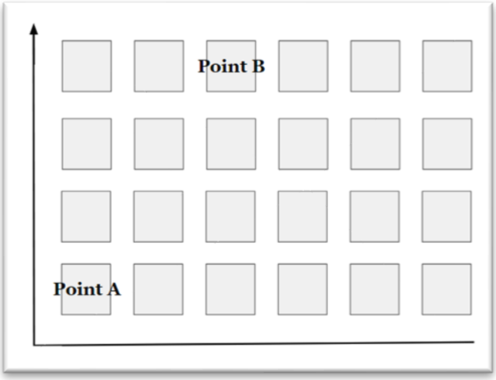
2.5

Kinematics

2D Motion

What do a waterfall, kicked soccer ball, and rollercoaster all have in common? Their motion needs to be described in two dimensions. While we can describe a car driving or a ball falling in a straight-line one-dimensional motion, all the previously mentioned examples move in two different directions that require two dimensions to describe. A waterfall goes outward and downward; a soccer ball that's kicked goes upward and outward. Let’s consider the following:

A person wants to go from point A to point B, but they can't move through the city blocks as there are buildings. Seeing how the city blocks are arranged, let’s set our axis and our origin in a point most convenient to us.



We can see that point A is 3 blocks away from point B horizontally and 4 blocks away vertically. This means that the person needs to travel 3 blocks on the x-axis and 4 blocks on the y-axis. But what if we want to find the displacement between point A and point B? Using the tip-to-tail method, we can place the tail of the vertical vector on the tip of the horizontal vector. Because our axes are perpendicular, then so are the vectors. What this means is that the angle is 90 degrees, and we can use the Pythagorean theorem to find the hypotenuse, or the total displacement of the person. Let's test it out:

$$x^2 + y^2 = d^2$$

$$\sqrt{x^2 + y^2} = d$$

$$\sqrt{(3\ blocks)^2 + (4\ blocks)^2} = d$$

d= 5 blocks, the person is displaced 5 blocks.

An important property of 2D motion is that each dimension is independent of one another. This means that anything that happens in one dimension does not affect the other dimension. This property is best seen when objects are in free fall. The object does not accelerate horizontally because gravity only affects the object in the vertical direction. If one object is launched horizontally and another is released from rest from the same height at the same time, then they will keep the same vertical position with each other throughout the entire motion despite differing horizontal position. Because of this principle, we can break apart 2D motion, allowing us to observe how the object moves in each dimension.

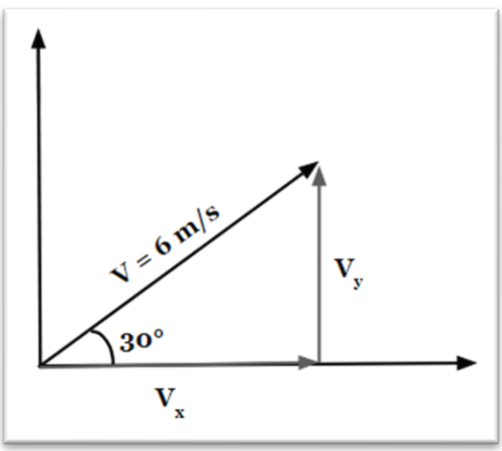
2.6

Kinematics

Projectile Motion

A projectile is defined as an object that experiences an initial thrust and then moves in freefall. Kicking a ball, launching a cannon, or rocks erupting from a volcano all are examples of projectiles. However, projectiles are rarely thrown just in the vertical direction, meaning we need to analyze the motion of the projectile in two dimensions.

Let's take the example of kicking a soccer ball at 6 m/s 30 degrees above our horizontal line, or the ground. Since the ball is moving both vertically and horizontally, we can split the motion into each part.



The vertical and horizontal vectors are perpendicular, meaning that we can take the initial velocity v and multiply it by the sine and cosine of the angle 30 degrees to find the velocity in each direction, which we will name v_x and v_y respectively.

$$v_x = v\cos(\theta)$$

$$v_y = v\sin(\theta)$$

In this scenario, v_x = $3\sqrt{3}$ m/s and v_y = 3 m/s initially. However, as the motion of the ball continues, the velocity in the y-direction will decrease due to gravity, while the velocity in the x-direction remains constant. This will result in one exact point where the ball is at the highest vertical position, which we can find with the kinematic equations.

$$v_f^2 = v_i^2 + 2a\Delta y$$

$$0 = v_y^2 - 2g\Delta y$$

$$2g\Delta y = v_y^2$$

$$\Delta y = \frac{vy^2}{2g}$$

$$\Delta y = \frac{(3m|s)}{2(10\ m/s^2)}$$

$\Delta y = 0.45\ m$ The reason we substitute 0 for the final velocity is that at the maximum height, gravitational acceleration has decreased the velocity where it is no longer moving in the positive y-direction.

2.6

Kinematics

Reference Frames

If you were in a train… then to an outside observer, you would be moving as fast as the train was. However, from your perspective, you aren't moving at all. This is a principle known as reference frames in physics. But what exactly are they? And how can we use them in physics?

We use reference frames in physics to describe the motion of objects and their relation to one another. A reference frame can be thought of as the perspective of the observer, which establishes a set of coordinates and reference points used to describe the motion outside of the frame. Using the previous example, the reference frame of an outside stationary observer to the train would see that the train and person are moving at a certain speed. However, the reference frame of the person inside the train will see that the train is moving at 0 m/s and the outside environment is moving in the opposite direction with the same speed.

The most commonly used reference frame is the "lab frame" — a stationary frame outside of any system. Think of it as the "fourth wall" on a TV show or video game.

Is it possible for a reference frame to be moving? Yes! For a moving reference frame, we need to consider its motion in the lab frame and apply that to everything outside of the new frame. If Boat A is moving to the right at 3 m/s and Boat B is moving to the left at 5 m/s in the lab frame, then from the reference frame of Boat A, Boat A is stationery and Boat B is moving to the left at 8 m/s. This is how we translate reference frames: we need to first translate it to the lab frame, and from there we can translate the lab frame to the desired reference frame!

Chapter 3

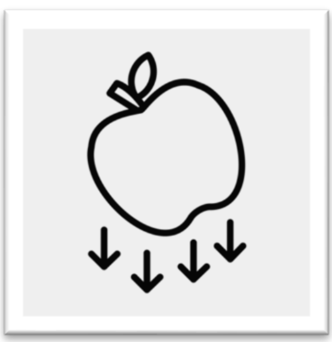
Dynamics

3.1

Dynamics

What is a Force?

You open the door to enter the store, you push instead of pull. Hmm... Why do some doors need to be pulled and others need to be pushed? Well to answer this question, we must learn what a force is! A force is essentially just a push or pull which causes something to have a change in its velocity or in other words, causes an object to accelerate. Isaac Newton created various laws for forces in the 1600s and we can use all of them to describe the movement of objects when mass is a factor! There are three of Newton's Laws which we will be learning and using throughout this unit so get ready to learn!



Question to Think About: What force causes the apple to fall? Hint: We have already discussed it before!

3.2

Dynamics

Newton's First Law

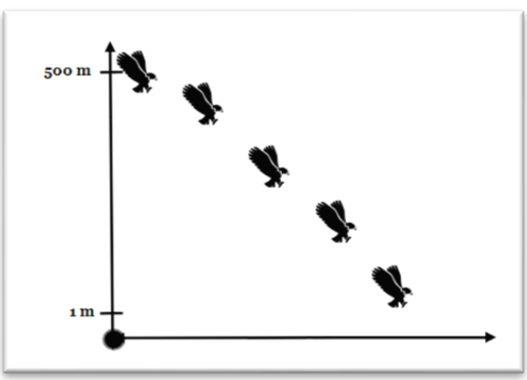
Have you ever wondered why objects which stay still, stay still? Yeah, it's a silly question, of course, if an object is staying still it will continue to stay still, but this is actually one of the most important questions when considering the concept of forces. This was first stated by Aristotle, an ancient Greek philosopher, but the concept was later refined by Newton. Newton completed the theory and created his first law:

Newton's First Law: "*An object in rest will stay in rest and an object in motion will stay in motion unless acted upon by an unbalanced force…*"

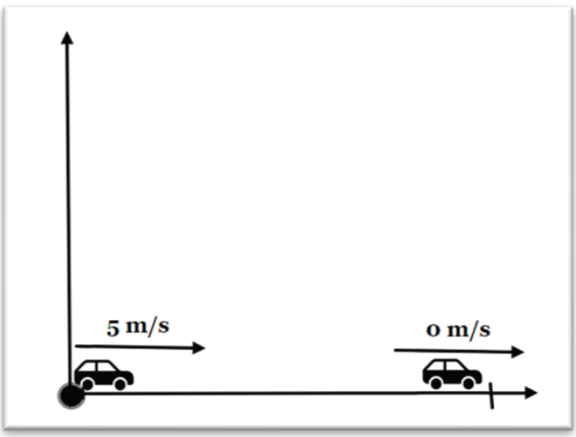
Well, what is this unbalanced force he mentions here? Essentially it is any force that can cause a change in an object's velocity, which is what we define as a force! However, a balanced force as we will learn later in the Equilibrium unit is a force that is balanced out by another force and thus doesn't change the velocity of an object. Let's understand Newton's first law through some examples.

Example #1: When a soccer ball is resting on the ground will it move if there are no forces applied on it? No, this is because an object at rest will stay at rest!

Example #2: A bird is gliding from 500 meters to 1 meter to catch prey, if the bird starts gliding will it continue to glide, or will its motion be stopped? The bird will continue to glide because an object in motion will stay in motion.



Example #3: A car going at 5 miles per hour continues to drive until it reaches its destination when the driver suddenly breaks! Will the car continue to move? No, it will not but this is odd, this doesn't follow Newton's First Law because an object in motion will stay in motion, but this is where the "unbalanced force" comes into play since there is a force applied by the breaks that cause the car to slow down to a stop, there is an unbalanced force and this is where Newton's Second Law can help to explain this situation!



3.3

Dynamics

Newton's Second Law and Mass vs Weight

Have you ever heard of the words "mass" and "weight" before and wondered if they were the same thing? Well, when we use these words in physics, they are not the same thing. When we use the word mass we are talking about the quantity or amount of matter an object has and we use kilograms (kg) to measure this value. So, for example, a typical car may be 2 tons which is 4,000 pounds or about 1,800 kg. This is what we usually think of as mass but if we use the same definition to define weight, we will find a problem. In physics weight is the measure of a force acting on an object due to the force of gravity. Yeah, that's right, gravity exerts a force, and it is measurable! We use Newtons (N) to represent weight or the force of gravity. But what exactly is a Newton? This is where Newton's Second Law comes in.

Newton's Second Law: $F = m\ x\ a$

What does this mean? Force is equivalent to mass times acceleration. We know that gravity has an acceleration known as "g" which is about 10 m/s^2 and objects inherently have mass. Let's say an apple is on Earth and weighs about 1 kg. What is its weight? It's super simple, all we do is multiply the mass (1 kg) by the force of gravity (10 m/s^2) which means the weight of the apple is 10 N or 10 Newtons. There are many different forces and remember anything is a force if it is a push or pull force that changes the velocity of an object. Let's use Newton's Second Law to solve some problems.

Example #1: A 5-kilogram bird is in free fall after jumping from a tree. The only force acting on it is the force of gravity, and it is currently on Earth. What is the bird's weight?

Weight: _______________ Newtons

Example #2: A balloon that weighs 2 Newtons on Earth is taken to Mars which has an acceleration due to gravity of about 3.72 m/s2. What is the balloon's mass and the balloon's weight on Mars?

Mass: _______________ Kilograms

Weight: _______________ Newtons on Mars

Example #3: John pushes a 15 kg box across a smooth surface with no friction. The box starts at rest but speeds up to 5 m/s after a displacement of 15 meters. What is the force John exerts on the box?

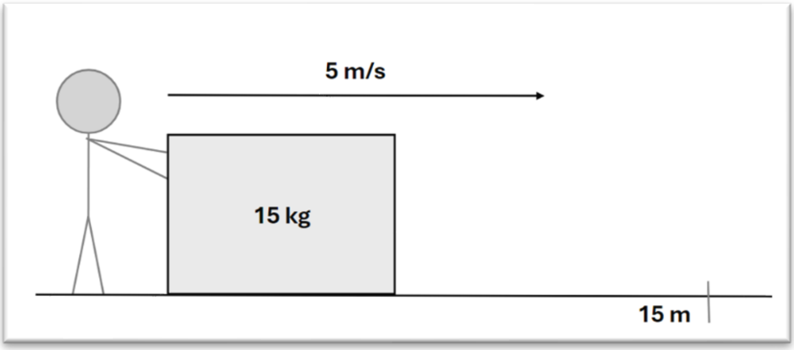
Force Exerted by John: _______________ Newtons

Answer Key:

Question #1: To answer this question let's consider the different parts of the question. We know the mass of the bird is just 5 kilograms and it is going through free fall, if we recall what free fall is, it is when the only force acting on a body is gravity so the acceleration of the bird will just be the acceleration due to gravity or "g" (10 m/s^2). If we use Newton's 2nd law $F = m\ x\ a$, we can plug in the mass and acceleration to get $F = (5\ kg)\ x\ (10\ m/s^2)$ which is 50 Newtons.

Question #2: When we solve this question we have to consider the difference between mass and weight, mass is something that is inherent in an object and doesn't change, you can be on Mars, on Pluto, or Jupiter, and the mass of the object will stay the same no matter what, but the weight will change due to the force of gravity on an object. To find the mass of the balloon we first consider its weight on Earth which is 2 Newtons. Since weight is equal to $F = m\ x\ g$ we can add the known values of F which is 2 N and g which is 10 m/s^2 on Earth. $2\ N = (m)\ x\ (10\ m/s^2)$ which gives us a mass of 0.2 kg. To find the weight on Mars we use the formula again but instead of 10 m/s^2, we will use 3.72 m/s^2 as that is the acceleration due to gravity on Mars. When we plug in the known values into the formula, we get $F = (0.2\ kg)\ x\ (3.72\frac{m}{s^2})$ and find the weight of the balloon on Mars as 0.744 N which is much less than the weight on Earth!

Question #3: For this question, a diagram would be very nice to represent the situation. Here is one below:



Here we will combine our kinematics knowledge and Newton's laws to answer the question. Since the equation for force is $F = m\ x\ a$ and we are missing the acceleration, we must use kinematics to help. Let's consider the parts we have, we know that the initial velocity of the box is 0 m/s because it begins at rest, the final velocity is 5 m/s, and the displacement is 15 meters. We can use the equation, $V_f^2 = v_i^2 + 2a\Delta x$. Let's plug in what we have, $(5\ m/s)^2 = (0\ m/s)^2 + 2a(15\ m)$. If we isolate the equation to find acceleration, we get a = 0.83 m/s^2. Now that we have everything we need for the equation we can plug in the values to find the force exerted. $F = (15\ kg)\ x\ (0.83\ m/s^2)$ and the force is equal to about 12.5 N!

3.4

Dynamics

Free Body Diagrams

Before we talk about Newton's Third Law, it is important to talk about Free Body Diagrams. In the third example for Newton's second law, we drew a diagram to represent the problem, and this was useful when we had just one force acting on the object, but what happens if there are multiple forces? Here we need to consider a free-body diagram also known as a "F.B.D". A FBD essentially shows all the forces acting on an object using arrows which are vectors of forces acting on a body. Making a FBD is simple and here are the steps.

Steps to draw a free-body diagram:

Step 1: Define the system and environment

Step 2: Draw a dot to represent the system

Step 3: When on Earth draw a pull from the Earth

Step 4: Find where the environment makes contact with the system

Step 5: Draw force vectors to scale on the systems dot for each point of contact

***Each force on the diagram must come from the object in the picture*

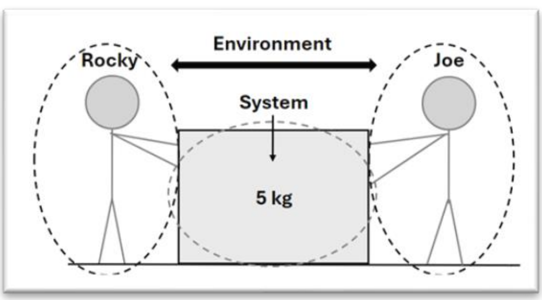
***The sum of the forces must equal to the mass times the acceleration*

The steps are simple but to understand them let's look at some examples. First, let's clear up a misconception. When we defined Newton's Second Law, we defined it as the equation $F = m\ x\ a$ which isn't incorrect but there is more to the equation than just that. The true equation is $\sum F = m\ x\ a$ which can be defined as Net Force is equal to mass times acceleration. But what exactly is a net force? A net force is the sum of all the forces on an object. This means that if multiple forces are acting on an object, the net force would be the sum of all the forces. Let's look at an example using FBDs to better understand this concept.

Example... Rocky pushes a rock across a flat surface with no frictional forces with a force of 100 Newtons but Joe also pushes on the same rock in the opposite direction as Rocky with a force of 50 Newtons. If the rock weighs 5 kg, find the acceleration of the rock.

Let's use our steps for FBDs and our new equation $\sum F = m\ x\ a$ to solve this problem.

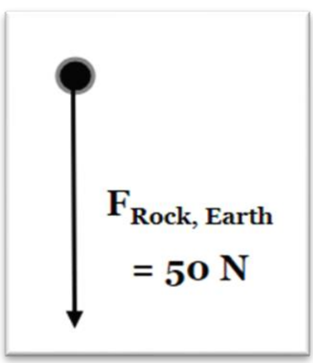
Step 1: Let's draw what the system and environment are.



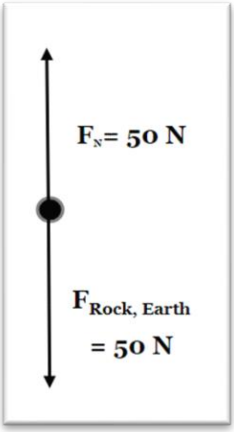
A system is the object/body which forces are acting upon, an environment is what causes the force. Here the system is the rock, and the environment is Rocky and Joe exerting forces on the Rock.

Step 2: For our FBD we use a dot to represent our system.

Step 3: Now we will add arrows to represent the pull from Earth which is better known as the Force of gravity. From now on we will be labeling our forces with subscripts (the small letter to the bottom right of the letter). The rule for labeling forces is $\mathbf{F}_{\text{"system","environment"}}$. This means we label a force by first writing the name of the system and then the environment which it comes from. So, for the force of gravity, we will label it as $F_{\text{Rock, Earth}}$. To find the magnitude of this force we simply use our equation $F = m\ x\ g$ and we get $F = (5\ kg)\ x\ (10\ m/s^2)$ and we get the value as 50 N. This force will almost always be drawn downwards towards the center of Earth. The force vector will be drawn to the center of the dot.

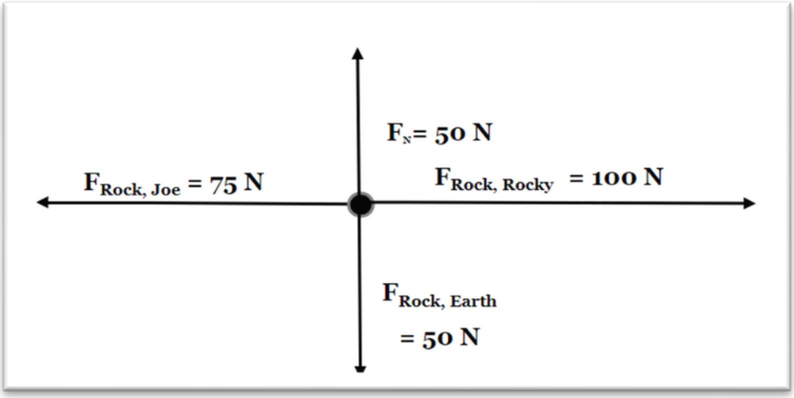


We aren't done yet though for this force when there is a contact force between two objects, here the ground and the Rock, we have something known as a "normal Force" which is a force that prevents an object from falling through a surface. In this situation, it is equivalent to the force of gravity but points upwards. It will be the same magnitude as the force of gravity but the opposite in direction. We can label this force as $F_{Rock,\ Ground}$ but we will just be writing F_N to simplify it.



Now we have all the forces in the y-axis, we can move on to the other steps which represent the forces in the x-axis exerted by the two people.

Step 4/5: Here we will add the forces from Rocky and Joe. Rocky exerts a force to the right of 100 N whereas Joe exerts a force of 75 N to the left. This means the force vector of Rocky should be slightly longer than Joe and both forces should be pointing in opposite directions.



Now that we have all the forces in the FBD we can move on to finding the net force exerted on the Rock and the acceleration of the rock. To add up all the forces we split up the forces based on the axis that they are exerted on similar to 2-D motion. We will have a $\sum F_x = ma_x$ and $\sum F_y = ma_y$.

This means we will add up all the forces in the x-axis and the y-axis separately. For the x-axis, we will add up the forces by the two people and we will also consider the direction they come from. Since the force from Rocky is in the positive x-axis it will be a positive force whereas the force from Joe since it is in the negative x-axis will be negative. This means...

$100\ N - 75\ N = (5\ kg)\ x\ a_x$. If we solve the equation, we get a_x = 5 m/s² and the net force in the x-direction is equal to 25N.

Now if we do the same thing for the y-axis, we can expect the acceleration to be 0 m/s² since the rock is only moving horizontally and not vertically so let's test this hypothesis.

$50N - 50\ N = (5\ kg)\ x\ a_y$. if we solve for a_y we see that it is equal to 0 m/s² which is exactly what we expected. We can even test this further in our FBD as if we did vector addition with the normal force and the force of gravity, we would see that both vectors cancel out!

FBDs may seem daunting and like a lot of work but with enough practice, they don't take longer than 30 seconds to set up. Let's try some more examples in the next page:

Try it on Your Own:

Example #1: A balloon that weighs 10 N on Earth is taken to Jupiter which has an acceleration due to the gravity of 23 m/s^2. It floats on this planet and has no other forces acting on it, draw its FBD and label all the forces acting on the balloon.

Example #2: A boxer punches a 50kg punching bag with a force of 2000 N, the punching bag is attached to a rope that holds it up, draw the FBD of the punching bag and the acceleration caused by the boxer.

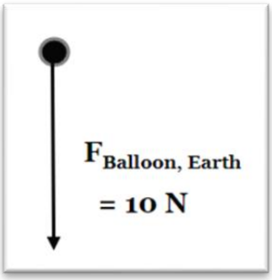
Acceleration: ______________ m/s^2

Example #3: A 200 kg car moves at a constant speed of 15 m/s on a frictionless horizontal surface when a truck suddenly attaches a rope behind the car and pulls with a constant force of 500 N from behind. Another rope is then attached to the front of the car which applies another force of 500 N to the car. What is the speed of the car while these forces are acting on the car?

Speed: ______________ m/s

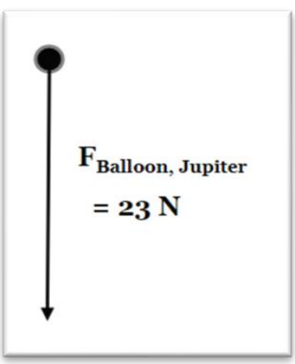
Answer Key:

Question #1: Let's draw a FBD for the Balloon when it is on Earth:

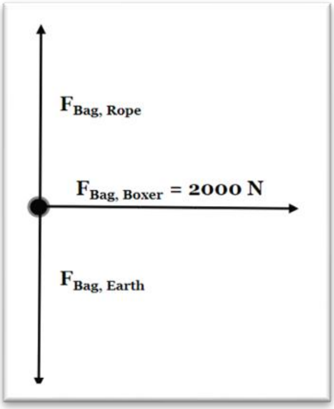


Since the balloon is floating and there is no point of contact with the ground or any other surface, we will not include a normal force. We can determine the mass of the Balloon using Newton's Second Law: $(10\ N) = m\ x\ (10\ m/s^2)$ this gives us a mass of 1 kg.

Now we can reuse Newton's second law to find the Forge of gravity when the Balloon is on Jupiter. $F = (1\ kg)\ x\ (23\ m/s^2)$, this gives us a force of gravity of 23 Newtons. The FBD is below:



Question #2: Let's begin solving this question by first drawing a free-body-diagram. We can determine the system to be the punching bag, and the environment to be the Earth, the punch from the boxer, and the rope holding it up. Let's label the forces in an FBD:



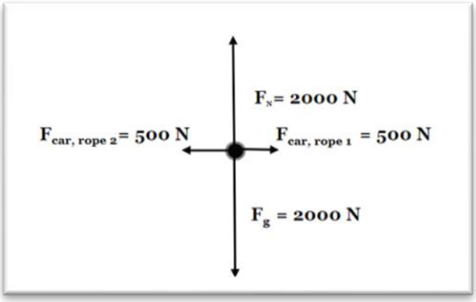
Now you may wonder why there is no force going leftwards if there is a contact force between the boxer's punch and the bag, there actually is but it is not a part of this system, this is something we will talk about more with Newton's Third Law but for now we will not include it into our system.

You also might wonder why the vectors for the force of gravity and the force from the rope are equal, this is because they must be the same size or else the bag will accelerate up or down. Since the bag is not moving up or down, we draw the vectors with equal magnitude. To find the actual value of these two forces we just use Newton's Second Law: $F = (50\ kg)\ x\ (10\ m/s^2)$ which gives us a force of 500 N for both vectors.

Now to find the acceleration for the punching bag we must find the forces in the x-axis which means we have to use this equation: $\sum F = m\ x\ a$. Let's include all the forces in the x-axis:

$2000\ N = (50\ kg)\ x\ a_x$ If we solve for a_x we get a value of 40 m/s^2 which is how fast the punching bag accelerates.

Question #3: Let's begin solving this problem by drawing a free-body-diagram. From now on, we will be labeling the force of gravity and normal force vectors as F_g and F_N because it is much simpler to write, but the best way to write these vectors is with the rules we stated previously.



We solved for the normal force and force of gravity by using Newton's Second Law:

$F = (50\ kg)\ x\ (10\ m/s^2)$ and got a value of 2000 N. Now to solve for the forces in the x-axis that cause the horizontal acceleration we must use our equation $\sum F_x = ma_x$. Let's add up the forces and solve for the acceleration $(500\ N) - (500\ N) = (200\ kg)\ x\ a_x$. If we solve for a_x we get a value of 0 m/s^2. But does this mean our car doesn't move at all? No, the acceleration will only provide a change in motion but if there is no change in motion, the motion of the object will stay the same and not change. Since in the problem it states that the car moves at a constant speed of 15 m/s it will continue moving at 15 m/s with no change in its speed. This is something known as kinetic equilibrium which we will be talking about in the next chapter!

3.5

Dynamics

Equilibrium

Have you ever been on a seesaw with a friend and you both equally balance each other out? Well, this is actually a form of equilibrium (we will talk more about the specific form in the torque unit). Equilibrium occurs when opposing forces balance each other out. We noticed this in example #3 in the Free Body Diagrams unit where the two forces pulling on the car balanced each other out. When we talk about equilibrium there are two forms, but one thing is held common between both:

"*In both kinetic and static equilibrium, the sum of the forces acting on the object is zero.*"

Static Equilibrium: An object is at rest and is not moving.

Kinetic Equilibrium: An object is in constant motion where the velocity stays the same.

Essentially what this means is that if the Net Force on an object is equal to 0 N, the acceleration of that object is 0 m/s^2, thus it can either be moving at a constant velocity which is Kinetic equilibrium, or it can be at rest which is Static Equilibrium. When determining if something is in equilibrium make sure that ALL forces in both the x-axis and the y-axis are balanced (the forces add up to 0 N and cancel each other out).

3.6

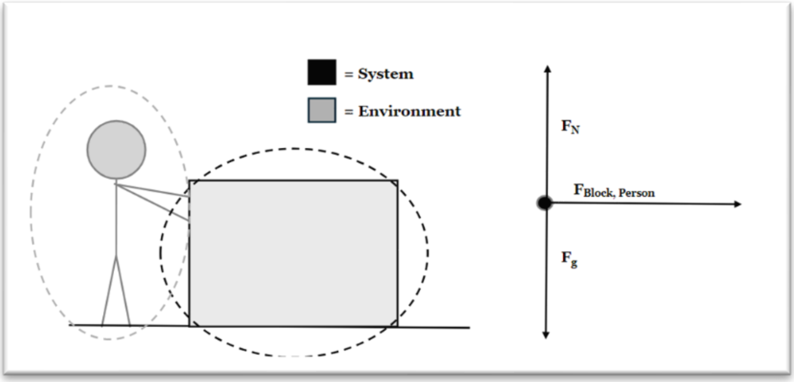
Dynamics

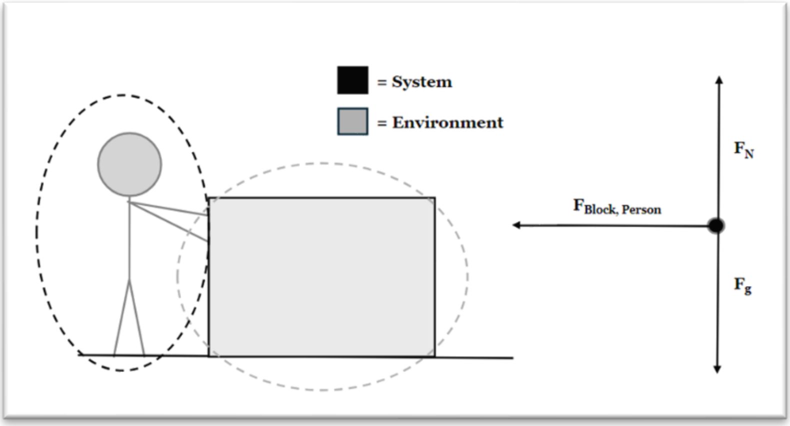
Newton's Third Law

When you punch something, have you ever wondered why you feel such a powerful force back on your fists? You may think that it's just how touching and punching objects works but have you ever thought that the wall punches you back? Would you be surprised if that is exactly how it works? Well, that is exactly how it works.

Newton's Third Law states that "*for every action there is an equal and opposite reaction*".

This basically means that if object A exerts a force on object B, then object B exerts a force of the same magnitude but opposite direction onto object A. These forces always come in pairs, here is a diagram to help show this. We usually use a FBD to represent these forces and it is super important to choose your object and system correctly to represent these forces.





As you can see the force applied to the block by the person is equal to the force of the block on the person. The direction of the force just changes based on what we chose at the system and environment.

Another very important thing to consider when completing problems with Newton's Third Law is that for a pair to be considered "third-law pairs", they must be from two different systems. This rule basically proves why the normal force, and the force of gravity are NOT third law pairs. Third Law pairs are usually marked with an "X" on the vectors to show that they are third law pairs.

Try it on Your Own:

Example #1: A person stands on a skateboard and pushes against a wall with a force of 50 N.

What is the force exerted by the wall on the person?

Force: ___________ Newtons

What happens to the skateboard because of the push? Explain using Newton's Third Law.

Example #2: Two people on ice skates face each other. Halim pushes Ahmed with a force of 30 N. Assume there is no friction between the skates and the ice.

What force does Ahmed exert on Halim?

Ahmed Exerts ________________ Newtons

If Halim has a mass of 60 kg and Ahmed has a mass of 75 kg, what are their respective accelerations?

Halim Acceleration ________________ m/s^2

Ahmed Acceleration ________________ m/s^2

Example #3: When a carpenter hammers a nail into a piece of wood, the hammer exerts a force of 100 N on the nail. Draw the FBD of the forces acting on the nail and the FBD for the forces acting on the hammer.

Answer Key:

Question #1:

1. This question is a simple example of Newton's Third Law, the answer is 50 N as we know that Newton's Third Law states that if Object A exerts a force on Object B, then Object B exerts the same force on Object A but in the opposite direction, this means the 50 N exerted by the person is exerted on the person by the wall.

2. As a result of the push, the skateboard will move backward. According to Newton's Third Law, the wall exerts an equal and opposite force on the person, causing the person (and the skateboard they are standing on) to move in the opposite direction of the push.

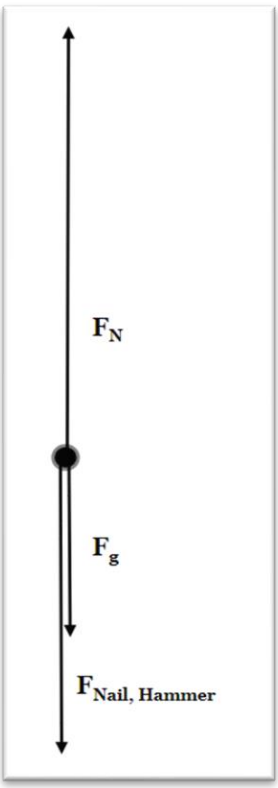
Question #2:

1. This is again another simple Newton's Third Law question, Ahmed would exert the same force as Halim but in an opposite direction, so he would exert 30 Newtons.

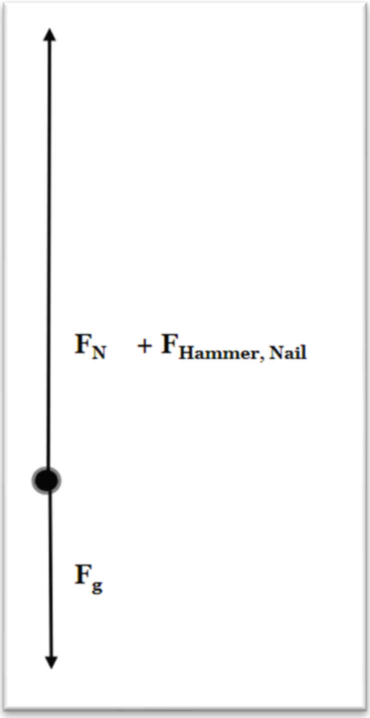
2. For this question we must combine our knowledge from Newton's Third Law and Second Law. Since we know they both exert 30 Newtons of force, we can use Newton's Second Law to find the acceleration of both people. Let's recall what Newton's Second Law is, $F = m\ x\ a$.

To find the acceleration of Ahmed let's plug in his mass $30\ N = (75\ kg)\ x\ a$ If we solve for his acceleration, we get 0.4 m/s². Make sure this is a positive acceleration as we can assume Halim pushes him to the right which is the positive x-axis. To find Halim's acceleration we will use a similar setup, but the force exerted by Ahmed would be negative since Newton's Third Law states that the force would be in the opposite direction so in this case it would be negative. $-30\ N = (60\ kg)\ x\ a$ If we solve for Halim's accleretaion we get -0.5 m/s²!

Question #3: First let's draw the FBD for the nail:



Here the force of gravity and the force on the nail by the hammer are in the same direction, we can add these vectors together and label them as one force, but we will keep it separate for now. The normal force increased due to the extra force from the hammer so we will draw it to match this. If the problem stated that the nail was accelerating downwards, we would make the normal force vector smaller. Now let's draw the FBD for the Hammer:



Here the vector for the normal force and the force of the nail on the hammer are labeled and combined. The vector is drawn to scale to represent the additional force. These are Newton's Third Law pairs so we can label them with an "x", but we will leave them without the "x" for this solution.

3.7

Dynamics

Friction

Have you ever rubbed your hands together on a cold winter day to warm up your hands? Have you ever pressed the brakes on your bike to slow down? Well, these are all examples of friction. Friction is considered a resistance that an object encounters when moving over a surface. There are two types of friction we will consider, and they are Static Friction and Kinetic Friction. Let's begin with the equation for friction. The general formula for friction is as followed: $F_F = \mu F_N$

The symbol "μ" is called "mu" and it is a coefficient which determines how much friction an object has. Normal force is also a part of this equation and multiplying both values gives us the value for the force of friction. Now let's identify the two different types of friction.

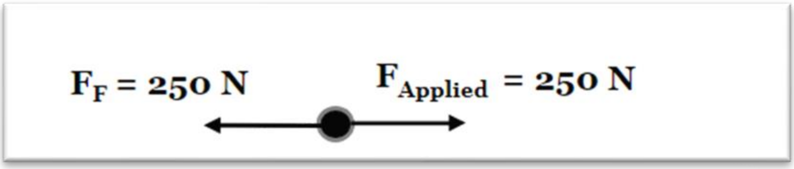
Static Friction: Is friction when an object is not moving, there is a different coefficient of friction for static friction than the other type of friction and the coefficient has a different subscript. For static friction the coefficient is "μ_s" which demonstrates that it is static. One thing to consider for the force of friction you get from this equation is that this value is the **MAX force of friction** an object can have before it turns into a different type of friction. Essentially, it's harder to begin moving an object because the max force of static friction which resists movement is greater than kinetic friction.

Kinetic Friction: This is the type of friction which occurs when an object is moving. Its value is less than static friction. Its coefficient is "μ_k" which demonstrates that it is kinetic. Let's look at an example to understand what this means.

Example: A 50kg rubber sled is sliding down a hill which is made of concrete. The static coefficient of friction for this sled and the surface is 0.9 while the kinetic coefficient is 0.68. Find the MAXIMUM force that can be applied to the sled so that it stays at rest.

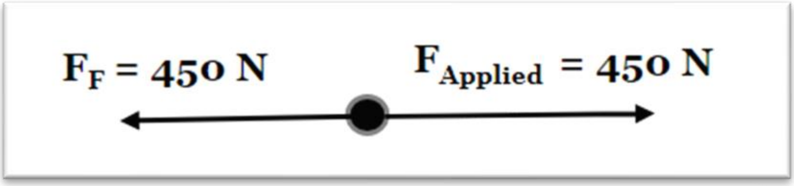
To answer this question, we will be using the equation $F_F = \mu_s F_N$ since we know that this gives us the value of the MAXIMUM static friction before an object turns into kinetic friction. To find the max static friction let's plug in the values. $F_F = (0.9)(50\ kg)(10\ m/s^2)$, we find the value to be 450 N. If the value of the force applied to this sled were to be any value greater than 450 N, then the friction would turn into kinetic friction and we would use the equation $F_F = \mu_k F_N$ instead, if we were to use that equation the value for friction would be much less as the kinetic coefficient is much smaller than the static coefficient which makes sense as an object in rest is harder to move than an object already in motion. Another thing to consider is that if the value of the force applied is less than 450 N, the value of the force of friction will be equal to the force applied as the two force vectors would cancer each other out. Here's a FBD to show these situations:

Force Applied < Max Static Friction:



Here the force applied is less than the Max Static Friction which we determined is 450 N, the force of friction and the applied force are equal which means the object is in equilibrium, since the object initially was not moving, it will continue to stay at rest.

Force Applied = Max Static Friction:

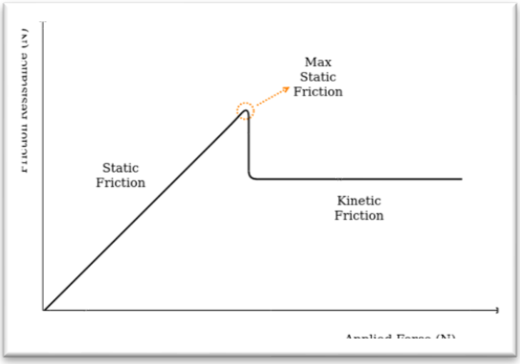


Here the applied force is exactly equal to the Max Static Friction which means it will still be in equilibrium, if the value of the applied force were to be any value greater than 450 N (the Max Static Friction) then the force of friction would turn into kinetic friction and that value is equal to $F_F = \mu_k F_N$ which we can determine by plugging in the values from the given information which means the kinetic force of friction is equal to $F_F = (0.68)(50\,kg)(10\,m/s^2)$ which is 340 N and this value is the value of the kinetic frictional force no matter how much force applied as long as that force is greater than 450 N.

Force Applied > Max Static Friction:

As you can see here the applied force is greater than the Max static friction and thus the force of friction is converted into kinetic friction which is less than the max static friction. The object would be moving as the applied force is greater than the force of friction, but the frictional force does slow it down as it is resisting the acceleration caused by the applied force.

Usually, we show this concept of static and kinetic friction through this graph:



Credit Research Gate

It clearly shows how the force of friction initially increases as the applied force increases, then after it reaches the max static friction value, as applied force is increased, the frictional force stays a constant value and is much less than the max static friction! Friction is a difficult concept to understand but it is definitely an amazing phenomenon. Let's take a look at some practice problems to refine our understanding of this concept!

Example #1: A wooden box weighing 30 kg is placed on a wooden floor. The coefficient of static friction between the box and the floor is 0.5. What is the maximum force that can be applied to the box without moving it?

Maximum Force: ________________ N

Example #2: A 20 kg metal block is sliding on an icy surface. The coefficient of kinetic friction between the block and the ice is 0.1. What is the frictional force acting on the block while it is moving?

Frictional Force: ________________ N

Examples #3: A 40 kg crate is at rest on a rough surface. The coefficient of static friction between the crate and the surface is 0.6, and the coefficient of kinetic friction is 0.4. If a horizontal force of 250 N is applied to the crate, will the crate move? If so, what will be the frictional force acting on it once it starts moving?

Will it Move:

☐ Yes: Frictional Force: ________________ N

☐ No

Answer Key:

Question #1: For this question we must find the Force of Max Static Friction, to do this we can simply just use our equation $F_F = \mu_s F_N$, let's plug in the known values $F_F = (0.5)(30\ kg)(10\ m/s^2)$, and we get a max force of static friction as 150 Newtons.

Question #2: In this question since we know that the metal block is already sliding on the ice, we can assume that the friction is kinetic. We can just simply plug in the known values into our equation $F_F = \mu_s F_N$, and we get $F_F = (0.1)(20\ kg)(10\ m/s^2)$, which is 20 Newtons.

Question #3: This question is a mix of everything we have learned about friction, first we must determine if the object is moving so we must find the max static friction. If we plug in the values to our equation, we get $F_{FMax} = (0.6)(40\ kg)(10\ m/s^2)$, and we get a maximum static friction force of 240 Newtons. Since our applied force is greater than that force (250 N > 240 N), the friction is actually kinetic so we must use the other equation to get our frictional force. $F_F = (0.4)(40\ kg)(10\ m/s^2)$, and the value of the force of friction is 160 Newtons. So yes, there is a frictional force, and its value is 160 Newtons. F_g

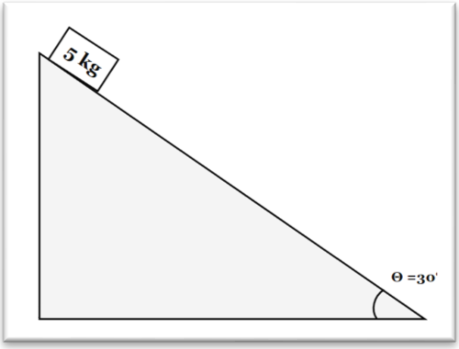
3.8

Dynamics

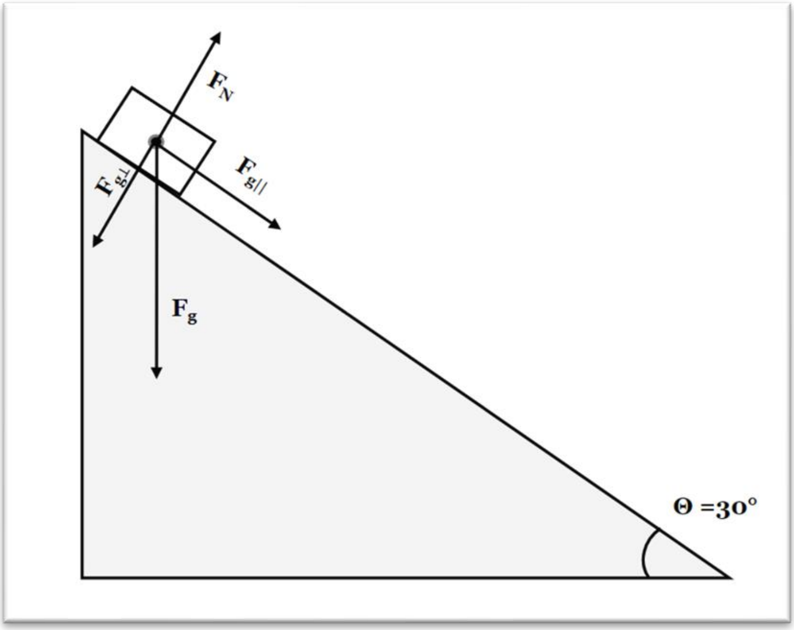
Inclined Planes

Have you ever gone down a steep hill on your bike, speeding up as go farther and farther down the hill? You may wonder if the laws of physics still abide in such a situation, well I'm here to tell you they do (albeit a little differently)! For us to understand inclined planes, ramps, hills, and anything where an object is at an angle, we must tilt our heads a little and see the world a little sideways.

Let's draw a diagram of what such a situation may look like below:



Here we have a 5 kg box on an inclined plane of 30°, if we were to find all the forces acting on the box, we would have to label two new axis which we will call the parallel axis and the perpendicular axis. We will use these two axes to show the components of the force of gravity and other forces acting on the box. Let's look at what this may look like:



As you can see, the force of gravity still points downwards towards the center of the Earth but now there are two other forces we have never seen. There is $F_{g||}$ better known as the force of gravity parallel to the surface which we talked about as the parallel axis. There is also $F_{g\perp}$ which is known as the force of gravity perpendicular because it is perpendicular to the surface. The normal force is pointing in the opposite direction of this force, and it is equal in magnitude. Now you may wonder what these two new forces are equal to and how they relate to gravity. Essentially, they are the two components of gravity and if we were to use the Pythagorean theorem to find their resultant, the magnitude would be exactly equal to the force of gravity. To find the values of these forces we use a simple formula to find their values. $F_{g||} = mgsin\Theta$ where Θ is the angle between the surface and the ground. And $F_{g\perp} = mgcos\Theta$. If these equations are hard to remember just remember this, "sine makes it slide" and we will see why in just a moment.

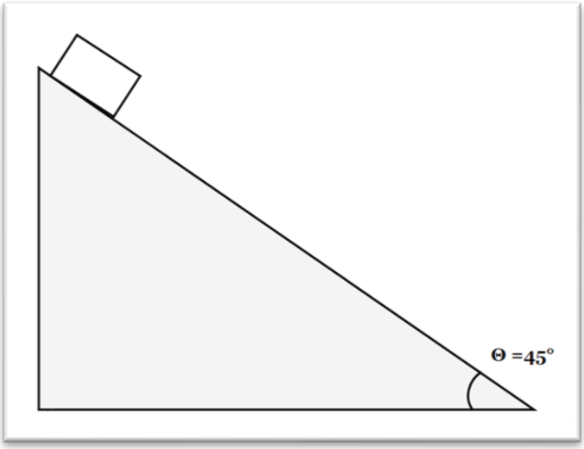
Since we know that the value of $F_{g\perp}$ is equal to F_N the equation for the normal force is equal to $F_N = mgcos\Theta$. Another thing to consider is that since the equation for friction also includes normal force it will also be changing because of an object being at an incline. The general equation for the force of friction (without upholding if it is static or kinetic) is equal to $F_F = \mu mgcos\Theta$. The force of friction's direction is dependent on whether the object is moving up or down, if the object is moving down the slope, then the friction will be upwards, but if the object is moving upwards, the force of friction will be downwards and acting with the Parallel force of gravity.

The acceleration of the object on the inclined plane will be in the parallel axis, this means that the force of the object which causes a change in its motion will be in that axis, when we consider other forces acting on the object we should always keep in mind that the forces should be added in this new axis rather the x-axis or y-axis as we usually would to solve for the acceleration of an object when multiple forces are acting on it.

Phew, that's a lot of new information, let's do some practice problems to understand inclined planes a little better.

Example #1: Let's say you have a 10 kg block moving downwards on an incline of 45°, if there are no external forces on the block and the coefficient of kinetic friction is 0.4, what is the acceleration of the block as it moves down the incline?

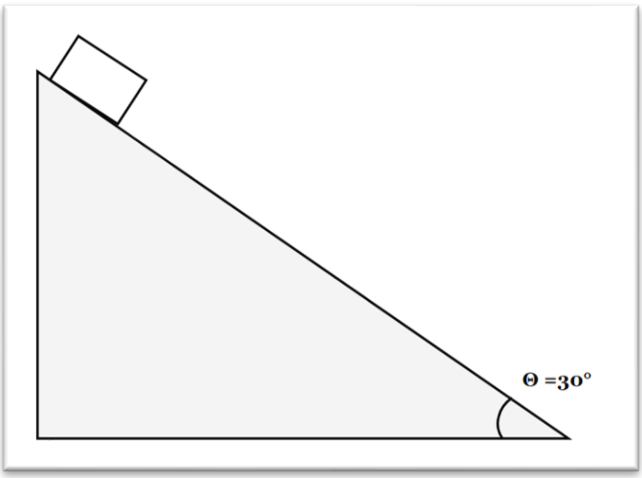
Draw a diagram to represent the situation and draw the forces acting on the block:



Acceleration of the block: ___________________ m/s²

Example #2: Alfred pushes a 12 kg block down an incline of 30° with a force of 10 Newtons. The coefficient of kinetic friction is 0.43, determining the acceleration of the block as it moves down the incline.

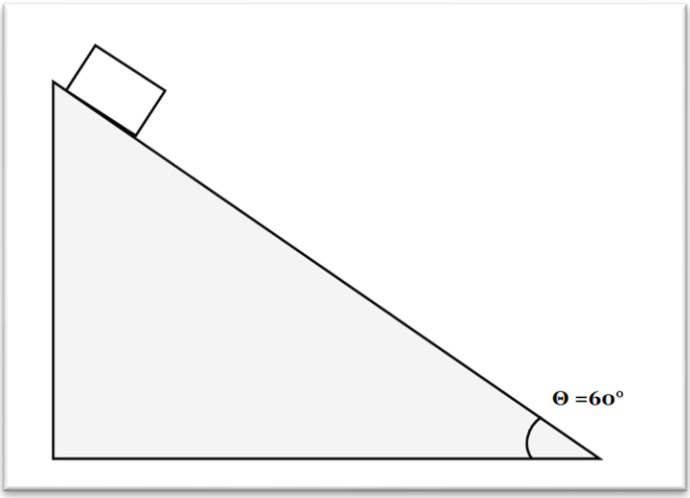
Draw a diagram to represent the situation and draw the forces acting on the block (remember that there is an additional force from Alfred!):



Acceleration of the block: _________________ m/s^2

Example #3: Dorian pushes a 1,000 kg car up an incline of 60° with a force of 5,500 Newtons, there is a force of friction in which the coefficient of kinetic friction is 0.2. Will Dorian successfully be able to push the car up the incline, if so, what is the acceleration of the car as it moves up the incline?

Draw a diagram to represent the situation and draw the forces acting on the block:

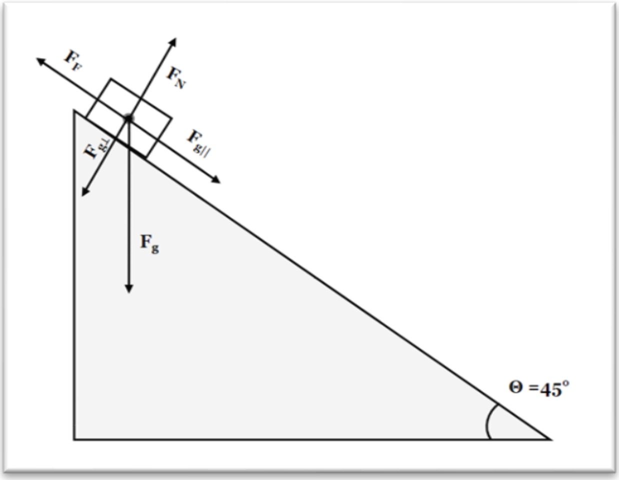


☐ No, he will not be able to move the car up the incline and its acceleration is ____________________ m/s^2

☐ Yes, he will be able to move the car and its acceleration is ____________________ m/s^2

Answer Key:

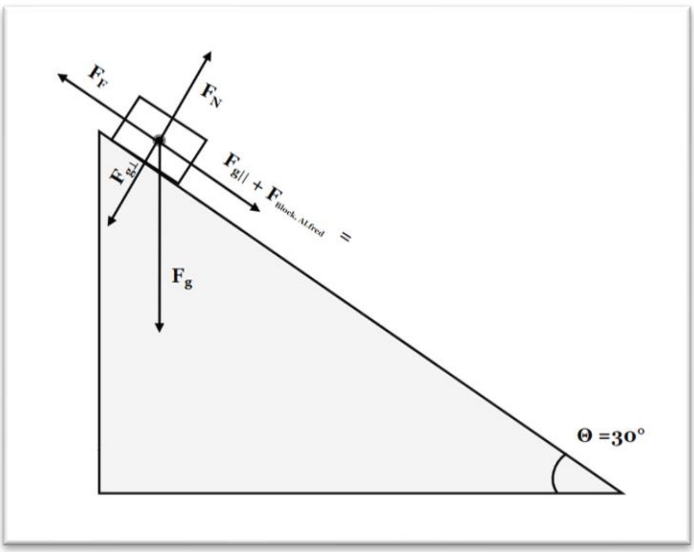
Question #1: To start answering this question let's fill in the forces acting on the system:



To solve for the values of these forces let's use our different equations, for the force of gravity we simply multiply 10 kg by 10 m/s² to get 100 N, for the parallel force of gravity let's use our equation $F_{g||} = mgsin\Theta$ if we plug in the known values we get $F_{g||} = (10\,kg)(10\,m/s^2)\sin(45°)$ the value is 70.71 N. To find the perpendicular force of gravity we use our equation $F_{g\perp} = mgcos\Theta$, if we plug in our known values, we get $F_{g\perp} = (10\,kg)(10\,m/s^2)cos(45°)$ which is equal to 70.71 N. Since this value is also equal to our normal force, we can also add that to our diagram. Now for friction since we already have the normal force, we must multiply it by the coefficient of kinetic friction which gives us the equation $F_F = (0.4)(70.71\,N)$, which gives us a value of 28.28 N. Now we have to use Newton's Second Law to solve for the acceleration.

Let's add up all the forces in which the acceleration occurs (the parallel-axis), remember that a positive acceleration is going down the parallel-axis. $(70.71\ N) - (28.28\ N) = (10kg)\ x\ (a)$. If we solve for the acceleration in the equation, we get a value of 4.24 m/s².

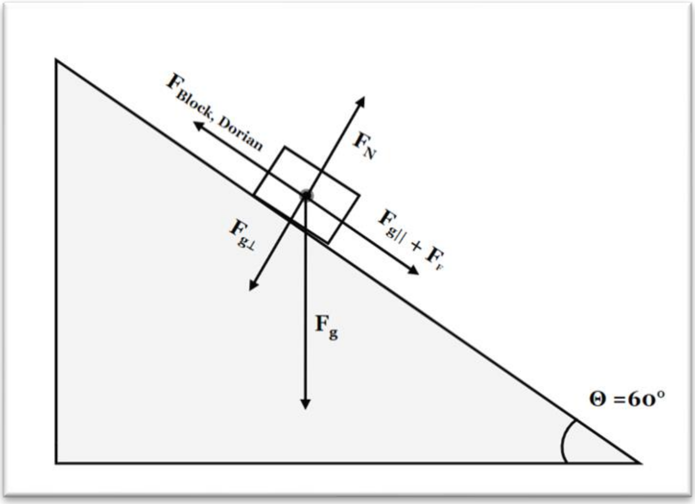
Question #2: To solve this question, we will be using a similar setup as the last question, let's first draw the FBD with all forces labeled:



The forces in this situation are similar to the previous problem but the parallel force of gravity is added to the force applied by Alfred on the block which makes it larger. I won't explain how to find each value because we explained how to do that in the previous question so take a look at that if you forgot! I'll list the values after we make our calculations. The force of gravity is 120N, the parallel force of gravity is 60N, the perpendicular force of gravity is 103.92 N, the normal force is also 103.92 N, the force of friction is 44.69 N and the force from Alfred is 10 N. Now again we will add all our forces using Newton's Second Law to find the acceleration.

$(103.92\,N) + (10\,N) - (44.69\,N) = (12\,kg)x(a)$. If we solve for acceleration, we get 5.77 m/s^2!

Question #3: Let's begin this question understanding the situation, the car will be moving upwards which means that the applied force will be upwards but the parallel force of gravity will be downwards meaning gravity actually slows down the car from moving but another thing to consider, since the applied force is upwards, the force of friction will actually be pointing in the same direction as the parallel force of gravity which is something that is really strange as it means the force of friction is actually positive! Let's draw the FBD to understand this better:



Let's figure out the values of all these forces, the force of gravity is 10,000N, the parallel force of gravity is 8,660.25N the perpendicular force of gravity is 5,000N which means the normal force is also 5,000N. The force applied by Dorian on the block is 5,500N and finally the force of friction is 1,000N. Let's use Newton's Second Law to figure out the acceleration of the car, (remember friction is positive in this example)

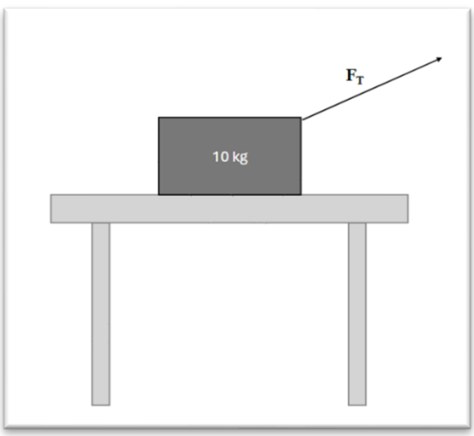
$(8{,}660.25\ N) + (1{,}000\ N) - (5{,}500\ N) = (1{,}000\ kg)\ x\ (a)$.
When we solve for acceleration, we get 4.16 m/s^2 which means that while yes, the car does move, it moves in the opposite way that Dorian pushes it which means it moves backwards. Physics really is cool, friction can help an object speed up, how interesting!

3.9

Dynamics

Tension and Hanging Systems

When we talk about tension in physics we usually think about strings and rope systems. Tension is a force that we call the force of tension but something very strange about this force is that there isn't one equation to represent it. Usually, the force of tension is a contact force similar to the normal force and thus just like normal force there is no specific equation for it and thus we must use Newton's Second Law to solve for it. Let's look at what tension may look like:



Let's assume the box is accelerating right at a speed of 2 m/s^2, there are no frictional forces, and the angle of the rope pulling on the box is 30°. Let's find the normal force of the object and the value of the force of tension.

First let's find the force of gravity which is simply just 100 Newtons. Now for the force of tension, we must break up the tension force into components similar to how we did with projectile motion. Let's find the force of tension in both the x-axis and y-axis.

In the x-axis the force is equal to: $F_{Tx} = F_T \cos\theta$

In the y-axis the force is equal to: $F_{Ty} = F_T \sin\theta$.

Since the only forces in the x-axis in this question is F_{Tx} as there is no friction, we can say that $F_{Tx} = m\ x\ a$.

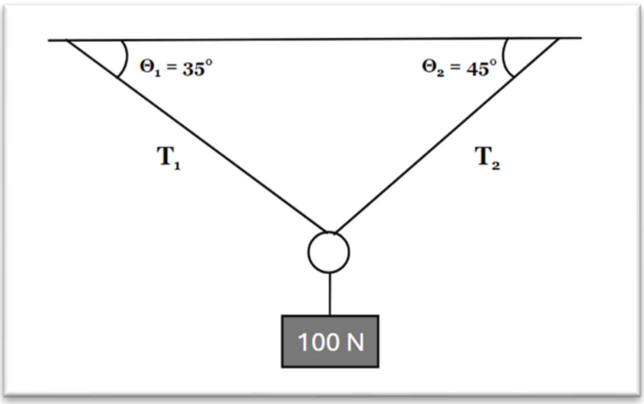
Knowing this information, we can plug in some values to solve for the force of tension, not just in the y-axis but the entire force $(F_T)(cos\ 30°) = (10\ kg)(2\ m/s^2)$, if we solve for F_T we get 23.09 N. Now that we have that value we can solve for the normal force. Since the rope is in the direction of north-east, it not only moves the box rightwards, but it also moves it up a little. To solve for the normal force, we must find all the forces acting in the y-axis. We know that there are three forces, the normal force, the force of gravity, and the force of tension in the y-axis. The force of tension will act upwards and so will the normal force which means they will be positive, and the direction of gravity is negative. Let's add these forces up and use Newton's Second Law to get our normal force.

$(F_T)(sin\ 30°) + (F_N) - (10\ kg)\left(10\frac{m}{s^2}\right) = (10\ kg)(0\ )$

we know that F_T is equal to 23.09 N so we can add that to the equation and solve for F_N which is 88.45 N. We used 0 m/s^2 as the acceleration as the box doesn't accelerate in the y-axis.

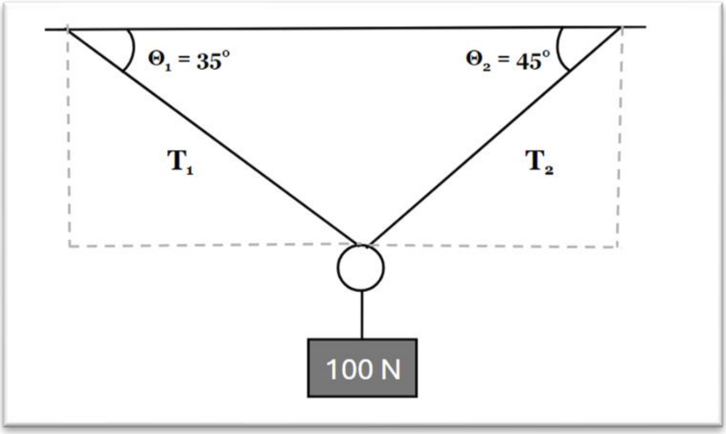
Force of tension questions can get crazy but with enough practice we can easily tackle these complicated questions!

Let's try another example of what a tension problem may look like, this one is a lot more complicated but uses the same principles as what we have already learned.



Find the values of T_1 and T_2.

This question might look very complicated but let's take it piece by piece: we know that all the forces in both the x-axis and y-axis will have an acceleration of 0 m/s^2 as the system is not moving. Another thing we know is that the forces in the y-axis are equal to 100 N which is the weight of the block. Let's split up the forces into their components:



Now that we have all the components labeled, we can create an equation that represents the different components. We will call the force of tension "T" in this situation just for simplicity.

For the forces in the y-axis, the equation would look like this:

$$(F_y) = T_1(sin\Theta_2) + T_2(sin\Theta_2) = 100\ N$$

For the forces in the x-axis, the equation would look like this:

$$(F_x) = -T_1(cos\Theta_2) + T_2(cos\Theta_2) = 0\ N$$

Let's find the $sin\Theta$ values and the $cos\Theta$ values:

$$T_1 sin(35°) = T_1(0.57)$$

$$-T_1 cos(35°) = -T_1(0.819)$$

$$T_2 sin(45°) = T_2(0.707)$$

$$T_2 cos(45°) = T_2(0.707)$$

If we add these values back to our two equations, we can see something very interesting:

$$(F_y) = T_1(0.57) + T_2(0.707) = 100\ N$$

$$(F_x) = -T_1(0.819) + T_2(0.707) = 0\ N$$

If this reminds you of a system of equations, that's exactly what it is! We can subtract both equations to remove the $T_2(0.707)$ so it makes finding the tension value much simpler.

$$T_1(0.57) + T_2(0.707) = 100\ N$$

$$-T_1(0.819) + T_2(0.707) = 0\ N$$

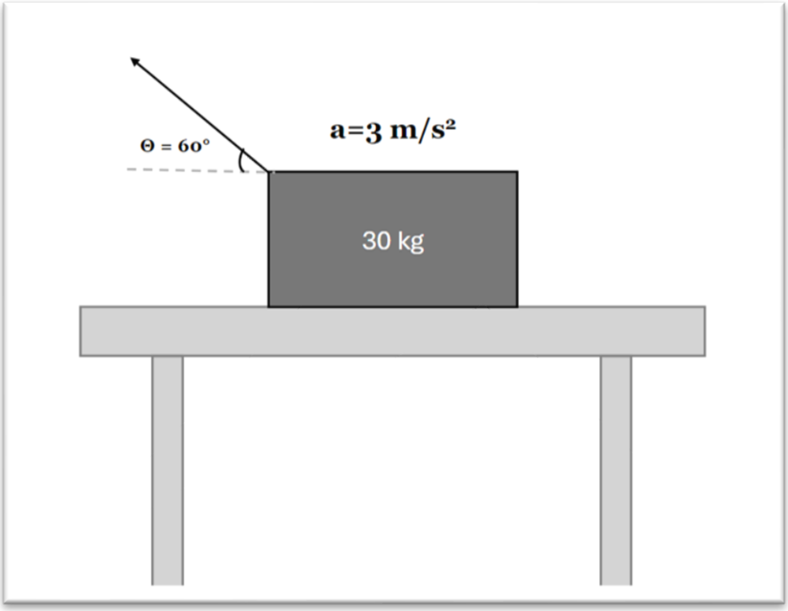
$$= \quad T_1(1.389) = 100\ N$$

If we solve for T_1 we get 72 N

Now if we plug in this value back into either equation and solve for T_2 we get a value of 83.4 N.

These tension problems require a lot of thinking, you won't always use a system of equations and may need to identify what exactly you might have to do to manipulate the equation to find the value you are looking for. Let's do some more practice problems to get better at this skill:

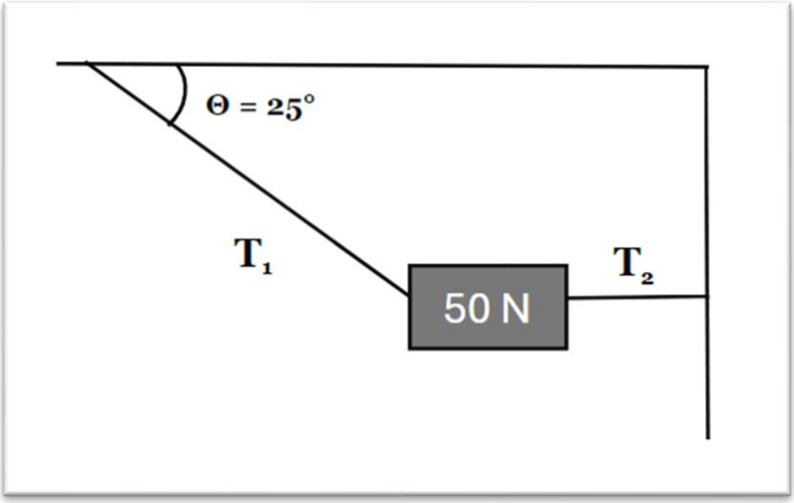
Example #1: A 30 kg bookbag sits on a table which is frictionless. If there is a rope attached to the bookbag which moves it leftwards at an angle of 60° and with an acceleration of 3 m/s2, what is the force of tension of the rope and the normal force acting on the bookbag?



Force of Tension on the Rope: ________________ N

Normal Force: ________________ N

Example #2: Find the force of tension on string 1 and string 2 which holds up a weight of 50 N and is attached by string 1 at a 25° angle and is attached horizontally with string 2. A diagram is below:



Tension One: _______________ N

Tension Two: _______________ N

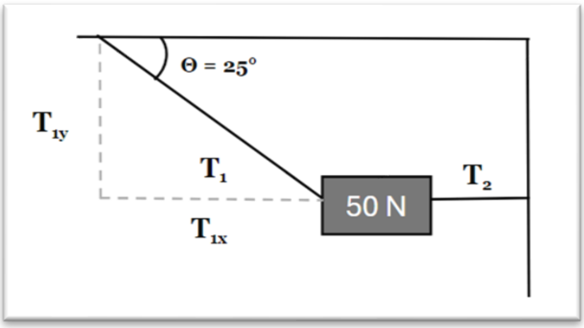
Answer Key:

Question #1: To solve this question, we must consider the two components of the string that provides the force of tension. We know that the bookbag moves at 3 m/s^2 towards the left, weighs 300 Newtons, and the string is at a 60° angle from the horizontal so we can use that information to find the force of tension. Let's use Newton's Second Law to solve this:

$\Sigma F_x = m\ x\ a_x$, if we add up the forces in the x-axis we get: $F_T\ cos(60°) = (30\ kg)x(3\ m/s^2)$ and if we solve for the force of tension, we get 180 N. Now if we want to find the normal force on the box, we have to find the difference between the force of gravity from the force of tension in the y-axis. We will simply use Newton's Second Law again to solve this: $\Sigma F_y = m\ x\ a_y$, since there is no acceleration in the y-axis the right side of the equation is just equal to 0. The only forces in the y-axis are the force of tension in the y-axis, the force of gravity, and the normal force so let's use information from the previous question to answer this question:

$F_N + F_T sin(60°) - (300\ N) = 0$. Since F_T is equal to 180 N we add that back into the equation: $F_N + (180\ N) sin(60°) - (300\ N) = 0$, when we solve for the normal force, we get about 144.12 Newtons which is what we could expect!

Question #2: Let's answer this question step by step, we know that to answer these types of question we have to break up the forces into their components so let's begin by doing that:



Let's create two equations, one for the forces in the x-axis and one for the forces in the y-axis, let's begin with the x-axis: $F_x = T_2 - T_1 cos\,\theta = 0$ the right side would be equal to 0 because the system is in equilibrium. For the forces in the y-axis, we get: $F_y = T_1\sin\theta - 50\text{ N} = 0$ the right side is similarly equal to 0 because the system is in equilibrium

From the first equation if we were to manipulate the problem, we can see that $T_2 = T_1 cos\,\theta$ so that means if we get the value of either T_2 or T_1 we can solve for the other tension. If we look at the second equation, we can see that $T_1 sin\,\theta = 50\ N$, if we plug in 25° for the angle, we can solve for T_1 which equals 118.31 N. If we plug that into the equation, $T_2 = T_1 cos\theta$ we see that $T_2 = (118.31\ N)\ cos(25°)$ and when we solve for T_2 we get 107.23 N! These problems look much harder than they actually are. Take time to analyze the problem and find the solution using the knowledge you have learned; you already know how to solve these problems so don't be scared to try and solve them!

3.10

Dynamics

Apparent Weight, Normal Force, and Elevators

Have you ever wanted to lose weight? Maybe you tried a diet, started exercising, well let me tell you the fastest and quickest way to lose weight. Well before I tell you the answer to this question, we must learn about what "apparent weight" is. Apparent weight is the reading on a scale which measures your weight and is the feeling of weight rather than your actual weight which is of course just mass times gravity. When you are on a scale it actually measures your normal force not your force of gravity. This means any change to your normal force is a change in the reading of a scale which technically isn't your real weight. Confusing huh? Yea it kind of is but there are many ways to change your normal weight so let's list out some examples!

Ways to change your normal force:

Jumping: Before jumping, as you push down against the scale to propel yourself upwards, this additional force temporarily makes your apparent weight greater than your usual weight. When you are in the air your normal force is 0 N, so your apparent weight is 0 and when you eventually land back down, as you decelerate, you exert an increased force on the scale which makes your apparent weight greater.

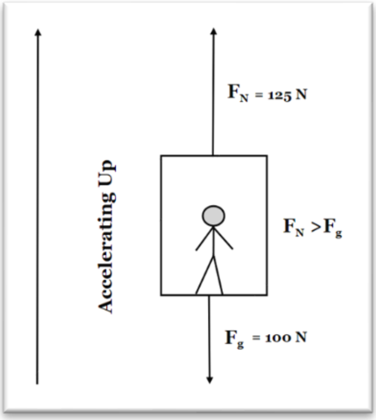
Inclined Plane: As we learned in the previous unit, when you are on an inclined plane your normal force is changed depending on the angle you are standing on, this is why scales usually tell you to measure your weight on a flat surface.

Elevators: This is the most interesting phenomenon which we will focus our studies on. On an elevator as you move upwards your apparent weight is decreased but as you move downwards, your apparent weight is increased, and we will learn why!

So, there are two scenarios in which your apparent weight is changed inside of an elevator, when you are accelerating upwards and accelerating downwards. Let's start with accelerating upwards.

When you accelerate upwards your acceleration is upwards which means the total force acting on you (the system) will be upwards. But we know that when we are standing on a surface with no forces acting on us, the only forces we have acting on us are normal force and the force of gravity so how would we represent this additional force going upwards which makes the elevator and us accelerate up?

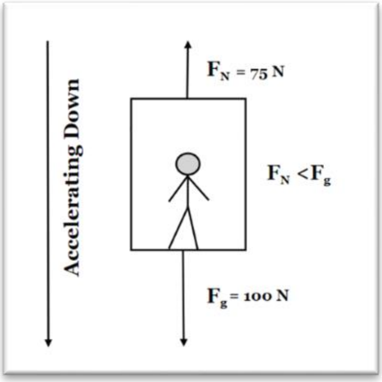
Well essentially what happens is that your Normal Force will change to represent this situation. Let's draw a diagram to help us understand:



In this diagram we can see a person standing in an elevator and the force of gravity of this person standing in the elevator is 100 Newtons so the person's mass is 10 kg, but something strange can be seen here. The person's normal force is greater than their force of gravity. Since we know that the elevator is accelerating upwards, we know that the force exerted is also upwards. Let's use Newton's Second Law to figure out what's going on.

$\Sigma F = m\ x\ a$ if we add up the forces in the y-axis, we get $(125\ N) - (100\ N) = (10\ kg)\ x\ (a)$. If we solve for acceleration, we see that it's equal to 2.5 m/s^2. This makes sense, the person is accelerating in an upwards direction so their acceleration must be positive. Essentially when a person or object is accelerating upwards in an elevator, their normal force must change to represent their acceleration. A similar phenomenon is seen when an elevator is moving downwards.

Let's look at an example to understand:



Here the same 10 kg person is on the elevator but now it is moving and accelerating downwards. The normal force is in turn smaller than the force of gravity. Let's use Newton's Second Law to explain this situation $\Sigma F = m\ x\ a$:

If we add up the forces in the scenario, we get: $(75\ N) - (100\ N) = (10\ kg)x(a)$, if we solve for acceleration, we see that it's equal to -2.5 m/s^2 and this is exactly what we expect. The normal force decreases to explain the object/person moving downwards.

Since we now know that we can change our apparent weight in an elevator, can we ever feel weightless in an elevator? Well, the answer to that question is yes but the only way to feel weightless is if the elevator is in free fall. This is because if we were to solve an equation to have a 0 N Normal force, we would have an equation that looks like this:

$(0N) - (F_g) = (m)\ x\ (a)$ we would get $-F_g = m\ x\ a$ and if we were to replace F_g with $m\ x\ g$ we would get $-m\ x\ g = m\ x\ a$ which simplifies to $-g = -a$, which basically means that the acceleration of any weightless mass must be equal to the acceleration due to gravity which means it must be in free-fall!

Let's try some practice problems:

Example #1: If Arjit stands in an elevator and has a weight of 160 Newtons but then his apparent weight changes because the elevator starts to move, and his apparent weight suddenly becomes 200 Newtons, what is the acceleration of the system and is it moving up or down?

□ Moving Up | Acceleration: ____________ N

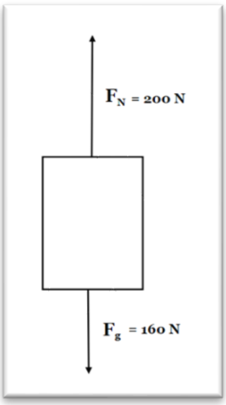
□ Moving Down | Acceleration: ____________ N

Example #2: If Yamal has a meeting and wants to go from the 5th floor to the 1st floor super-fast, accelerating downwards at -8.2 m/s^2, what would his apparent weight be during the ride if he normally weighs 120 Newtons?

Apparent Weight: ____________ N

Answer Key:

Question #1: To begin solving this question we should draw a model of the situation:

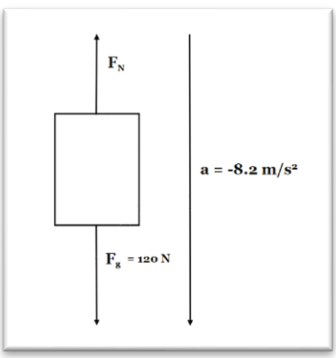


The problem is asking us to find the acceleration on Arjit and the elevator. We can simply just use Newton's Second Law to solve this problem, let's add up the forces and solve for the acceleration:

$$(200\ N) - (160\ N) = (20\ kg)(a)$$

If we solve for acceleration, we get a value of 2 m/s² and since this value is positive, Arjit must be moving upwards.

Question #2: Let's begin solving this problem by representing the situation with a diagram:



To answer this question, we must find what the question is asking for. The question is asking for the Normal force acting on Yamal. We are given the acceleration of the system and the weight so we can simply use Newton's Second Law to solve for the Normal Force:

$(F_N) - (120\ N) = (12\ kg)\,(-8.2\ m/s^2)$. If we solve for the normal force we get an apparent weight of 21.6 Newtons, wow elevators really can make us lose (apparent) weight quickly!

3.11

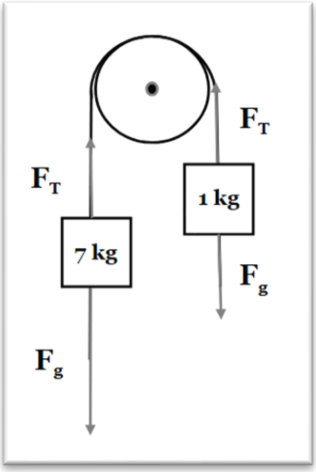
Dynamics

Atwood Machines and Pulleys

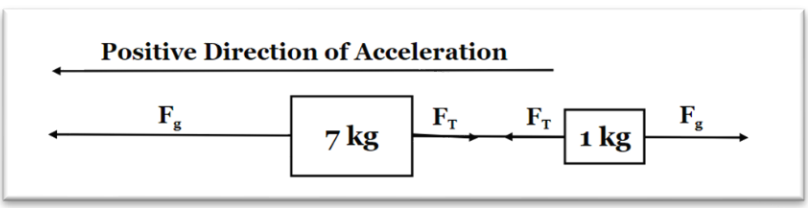
Have you ever used a rope and pulley to move something up and down easily? The way a vertical pulley works is when a force pulls one side of the pulley down, the other side is moved up. In this topic we will be studying the physics behind how this works and how to find specific parts of this scenario. We sometimes call these problems Atwood machine problems, but that's just a fancy name for a pulley. Let's look at what some of these problems could look like:

There is a 7 kg weight on one side of an Atwood machine and a 1 kg weight on the other side of an Atwood machine. Find the force of tension on the string holding both the weights. Assume there is no friction, the pulley and the pulley and string are massless.

Let's begin answering this question by drawing a diagram of the situation:



We can see that the Force of tension acts upwards for both weights and both weights have a force of gravity acting downwards. The best way to find the force of tension is to do this strategy which I like to call "straightening out the system". Essentially what we want to do is bend the system to make it straight and horizontal. We will define the positive direction for the acceleration of the entire system to be in the direction which the net force acts, in our scenario since the 7 kg will move downwards and provide the direction for the net force, the positive direction will be counterclockwise and towards the left when we straighten the system as shown below:



As you can see, all we did was in a way "unbend" the pulley system, so it looks like something a lot easier to solve. Now to solve this question we must understand the difference between internal forces and external forces, the force of tension will be an internal force because it is something that is internal to the system, it is inside of the system and is not a force coming from an external force. Something like the force of gravity is an external force since it's coming from outside of the string-pulley system. We will solve the system separating external forces from internal forces. To find the acceleration of the system we will be using external forces to solve for the acceleration. We will be using Newton's Second Law to solve this $\Sigma F = m \ x \ a_{system}$. Let's add up the external forces in the x-axis (technically the y-axis) and we have to remember which direction is positive and which is negative, leftwards is positive and rightwards is negative:

$(70 \ N) - (10 \ N) = (7 \ kg + 1 \ kg) \ x \ a_{system}$. We can now easily solve for the acceleration of the system which is equal to 7.5 m/s². Now that we have the external forces solved and have the acceleration of the system, we can now solve for the internal forces which are the tension forces. To solve for the tension all we have to do is find the force of tension on either the 7 kg mass or the 1 kg mass system as we know tension is always the same in a system. Let's solve for the tension in the 7 kg mass system first.

Let's draw the FBD for the 7 kg system first while straightened out using information we previously learned:

As we can see here this is simply just a Newton's Second Law problem for us to solve

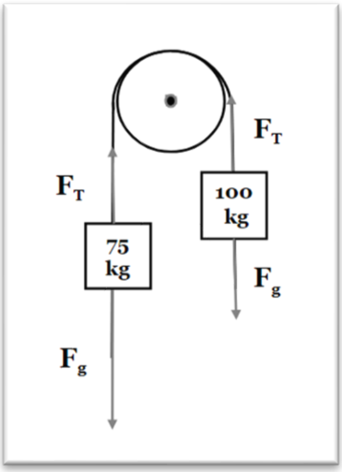
$\Sigma F = m \ x \ a_{system}$ if we add up the forces (making sure the numbers are correctly positive of negative):

$(70 \ N) - \ (F_T) = (7 \ kg) \ x \ (7.5 \ m/s^2)$, when we solve for the force of tension, we get a force of tension of 17.5 N. To prove that you can use either weight to solve the force of tension let's do the same thing using the other weight.

$\Sigma F = m \ x \ a_{system}$ remember to correctly identify if each force is negative or positive.

$(F_T) - \ (10 \ N) = (1 \ kg) \ x \ (7.5 \ m/s^2)$. If we solve for the force of tension, we again see that we get 17.5 N! Now that we learned how to solve Atwood problems let's do some practice problems:

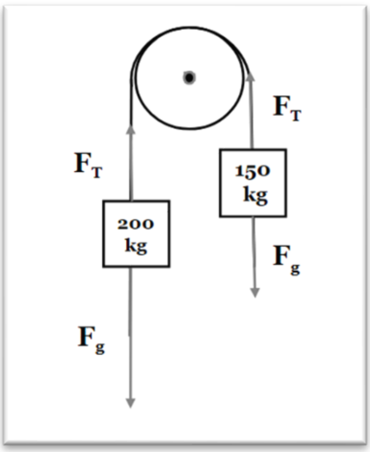
Example #1: There is a 100 kg weight on one side of an Atwood machine and a 75 kg weight on the other side of an Atwood machine. Find the force of tension on the string holding both the weights. Assume there is no friction, the pulley and the pulley and string are massless. Find the acceleration of the system and the force of tension between the strings.



Acceleration of the System: _______________ m/s^2

Tension between the strings: _______________ N

Example #2: Ricky wants to use an Atwood machine to lift a heavy boulder from his driveway, Ricky weighs 200 kg and the rock weighs 150 kg. Find the force of tension on the string holding both Ricky and the boulder. Assume there is no friction, the pulley and the pulley and string are massless. Find the acceleration of the system and the force of tension between the strings.

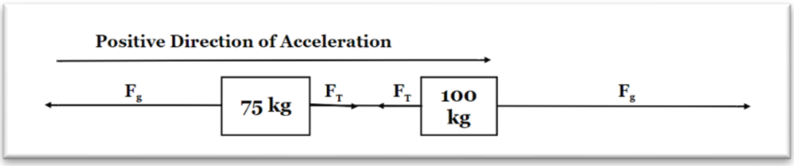


Acceleration of the System: _______________ m/s^2

Tension between the strings: _______________ N

Answer Key:

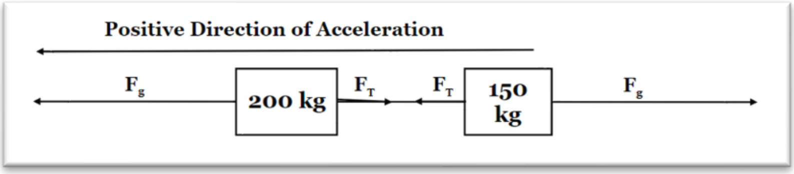
Question #1: To answer this question the first thing we must do is identify the direction of the net force to determine the positive direction of acceleration, in this case since the 100 kg is heavier than the 75 kg, the system's net force will be clockwise which means the positive direction of acceleration is towards the right when we straighten out the system, let's straighten it out:



Now we can solve for the acceleration of the system, let's set up an equation using Newton's Second Law: $(1000\ N) - (750\ N) = (75\ kg + 100\ kg)\ x\ (a_{system})$. When we solve for the acceleration of the system we get 1.43 m/s^2. Now that we have the acceleration let's find the tension by using either the 75 kg system or the 100 kg system. Let's use the 100 kg system but we will get the same answer if we use the 75 kg system as well. Let's set up the equation to find the force of tension:

$(1000\ N) - (F_T) = (100\ kg)\ x\ (1.43\ m/s^2)$, when we solve for the force of tension we get about 857.14 N.

Question #2: Let's begin answering the question by considering the same things as we did for the previous problem, the acceleration of the system will be counterclockwise and towards the left side when we straighten out the system, this means that the positive direction of acceleration will be leftwards. Let's straighten out the system:



Now that we have the system straightened out, we can solve for the acceleration of the system:

$(2000\ N) - (1500\ N) = (200\ kg + 150\ kg)\ x\ (a_{system})$, when we solve for the acceleration, we get 1.43 m/s^2. Now let's solve for the force of tension using the 150 kg boulder system:

$(F_T) - (1500\ N) = (150\ kg)\ (1.43\ m/s^2)$, when we solve for the force of tension, we get 1714.29 N!

Now you know how to solve Atwood machine problems!

3.12

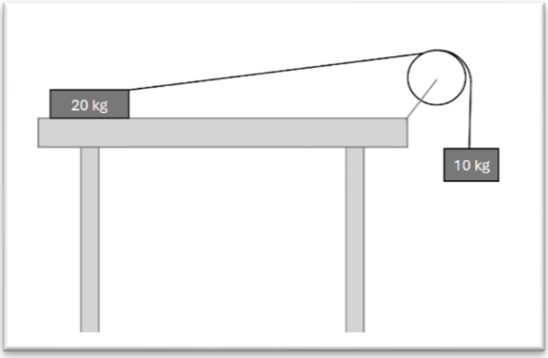
Dynamics

Modified Pulleys

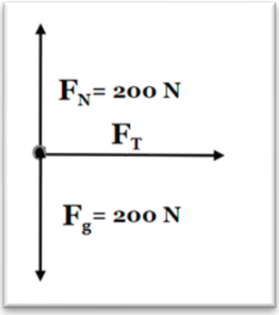
We recently learned about Atwood machines but now let's learn about a different rendition of the Atwood machine... Drum roll please... The modified pulley. It's revolutionary! The modified pulley has one weight on a surface which can only move horizontally, and another weight off the surface which moves vertically upwards or downwards, this means that the pulley off the surface will cause the acceleration of the weight on the horizontal surface. Let's look at an example to understand the situation:

Frank places a 20 kg heavy metal block on a frictionless table and attaches the block to a string on a pulley which is being pulled down by a 10 kg metal block. Find the acceleration of the system and the force of tension on the string.

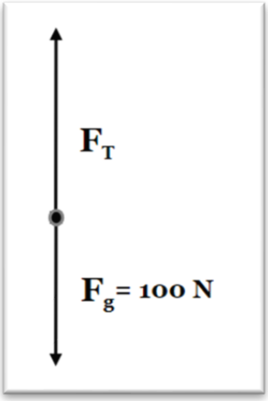
Let's draw a diagram of the problem:



Modified pulley questions are pretty similar to Atwood machine problems and hanging system problems so we can use our knowledge from previous units to understand how to solve these types of questions. The first thing we will be doing as always is creating a FBD, let's make one for the 20 kg block system and one for the 10 kg block system. Let's start with the 20 kg block:



Now for the 10 kg block system:



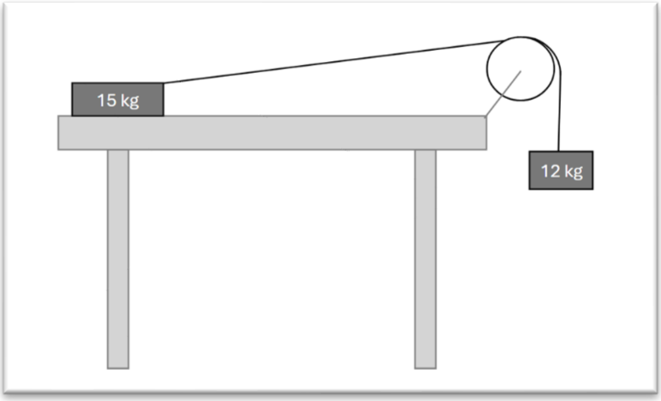
We can notice that the force of tension is a Newton's Third Law pair, and it is equal in both systems even though it is facing two different directions. To solve for the force of tension we can have to find the acceleration of the system, since the acceleration is going to be downwards caused by the force of gravity by the 10 kg block the only force causing the net force will be the 10 kg block's force of gravity. The 20 kg block will not be a part of the net force except for its mass because it is sitting horizontal and not causing a force in the y-axis. We can set up the equation as such: $(100\ N) = (20\ kg + 10\ kg)\ x\ (a_{system})$, when we solve for the acceleration, we get about 3.33 m/s^2. Now that we have the acceleration of the system, we can solve for the tension just like we did for previous Atwood machine problems. We can use either system to solve for the force of tension, let's use the 20 kg block system. Let's set up the equation like this:

$F_T = m\ x\ a_{system}$ and when we plug in the value, we get $F_T = (20\ kg)\ x\ (3.33\ m/s^2)$ and we get a force of tension of 66.67 Newtons. When we do the same for the other system, we set up an equation that looks like this: $F_g - F_T\ =\ m\ x\ a_{system}$ and when we plug in the values we get $(100\ N) - F_T\ =\ (10\ kg)\ x\ (3.33\ m/s^2)$, this gives us a force of tension of 66.67 Newtons just as we would expect.

** Remember that we consider the positive direction of acceleration as the direction which the net force goes towards, this is super important for problems like these! **

Let's do a practice problem to understand this topic better:

Example #1: There is a 15 kg cart on top of a frictionless table which is connected to a weightless rope on a pulley attached to a 12 kg bookbag. Find the force of tension on the rope and the acceleration of the system?

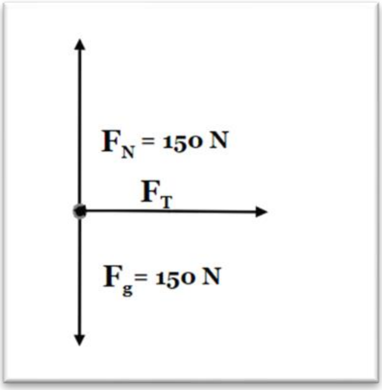


Acceleration: _______________ m/s²

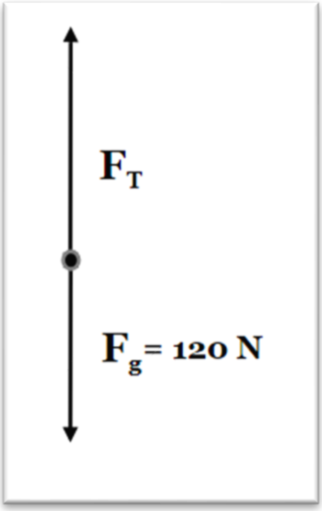
Force of Tension: ______________ N

Answer Key:

Let's begin answering this question by creating the FBD for both the 15 kg cart system and the 12 kg book bag system, let's start with the 15 kg system:



And for the 12 kg system:



To solve for the acceleration of the system we have to consider what causes the net force, in our situation it's the force of gravity of the 12 kg bookbag. Let's set up our Newton's Second Law equation:

$(120N) = (12\ kg + 15\ kg)\ (a_{system})$, when we solve for the acceleration of the system, we get a value of 4.44 m/s². Now let's solve for the force of tension, as we know we can use either system. Let's use the 15 kg system: $(F_T) = (15\ kg)\ x\ (4.44\ m/s^2)$, when we solve the equation for the force of tension, we get 66.67 N!

3.13

Dynamics

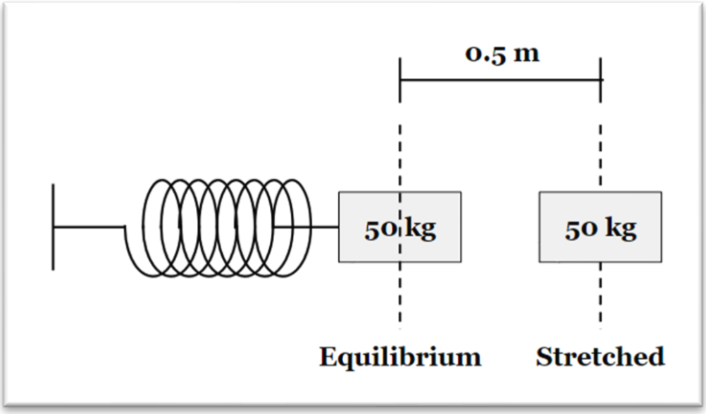
Spring Force

Boing, boing, boing, have you ever used a slinky and noticed the way it stretches and unstretches? Well, we can actually measure the force from this spring and in later units we can learn the energy that this spring contains! The formula is simple, we call it Hooke's Law:

$F_s = -kx$ essentially what this equation means is that the spring force is equal to the distance a spring stretches or compresses (x) multiplied by a constant (k), we call this constant the "spring constant" which should come as no surprise. This force is negative because it works to restore a string back to its equilibrium position where there is no compression or extension. The units for the spring constant are (N/m) and they represent how "stiff" or "stretchy" a spring is, it tells us how much force is needed to be applied to a spring to stretch or compress it 1 meter. Let's take a look at an example:

Example… Raymond stretches a spring that has a 50 kg block attached to it. The spring constant is 400 N/m and Raymond stretches it a distance of 0.5 meters. Find the spring force provided by the spring.

Let's answer this question by first drawing a diagram of this situation:



When the spring is in equilibrium the spring force is equal to 0 N, but when it is stretched or compressed there will be a spring force. To solve for how much this force is equal to we will just simply use the equation and plug in the known values:

$F = -(400\ N/m)\ (0.5\ m)$, we get a spring force also known as the "F_s" which is equal to -200 Newtons, the value is negative to let us know that this force will act leftwards to make the block move leftwards to its equilibrium position. The value of the mass attached does not matter! Let's practice with some practice problems:

Example #1: Juan attaches a 10 kg mass to a vertical spring with a spring constant of 250 N/m. The spring stretches until it reaches its equilibrium position. What is the displacement of the spring from its original position?

Displacement: _______________ meters

Example #2: Anto compresses a horizontal spring with aspiring constant of 200 N/m a distance of 0.4 meters. What is the magnitude of the force required to hold the spring in this compressed position?

Force: _____________ Newtons

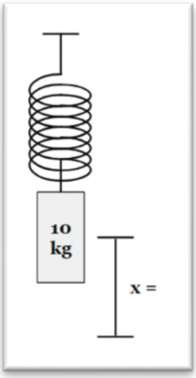
Examples #3: Aiden stretches a spring with an unknown spring constant 0.25 meters which exerts a force of 50 N. Find the spring constant.

Spring Constant: ______________ N/m

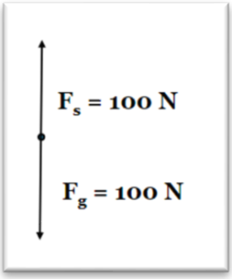
Answer Key:

Question #1:

Let's answer this question by first drawing a diagram to represent the situation:



We are basically trying to find when the force of gravity by the mass will be balanced by the spring force. This is what we call equilibrium. Let's draw a FBD of what that would look like:



As we can see, we are trying to find when the spring force is equal to 100 N which balances out the force of gravity and makes the spring achieve equilibrium.

Since F_s is equal to 100 N and we know the spring constant is equal to 250 N/m, let's just plug in the known values to find the displacement.

$(100N) = -(250\ N/m)\ (x)$, if we solve for the value of x, we find that it is equal to 0.4 meters.

Question #2: This question is pretty straightforward, we are essentially just trying to find the spring force and we have all the given information needed to find that, let's plug in the known values into our equation: $F_s = -kx$, when we plug in the values we have we get F_s = -(200 N/m)(0.4 m) and we get a spring force of -80 Newtons.

Question #3: To solve for the spring constant we can just use our spring force equation and solve for our missing variable: $F_s = -kx$, let's plug in what we have: $(50\ N) = -k(0.25\ m)$ when we solve for the spring constant value "k" we get a value of 200 N/m. Spring force equations are pretty easy and straightforward!

3.14
Dynamics

Universal Law of Gravitation

When an apple falls from a tree, we learned that a force called gravity causes it to fall and accelerate until it hits the ground. Well, what about the gravity between two planets? We can't just use our regular equation $F = mg$ in this situation because we don't have the acceleration due to gravity. Hmm…. What should we do? Well Sir Isaac Newton thought of that, he created an equation known as the Universal Law of Gravitation to find the force of gravity between any two objects whatever distance away from each other. The equation is:

$F_g = G\frac{m_1 m_2}{r^2}$, this equation essentially states that if you multiply the masses of two objects and divide it by the square of the distance between each of their centers of gravity, and finally multiply by a constant "G" a.k.a "Big G", we can get the force of gravity between both objects. The value of "G" was calculated after Newton's death by Henry Cavendish who found the value to be a very specific number which is ($6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$). Yea this is a very specific number with a bunch of units but luckily for our case we won't need to memorize the units because most of them will cancel out when we do our multiplication and division! Gravity is pretty interesting, the force of gravity felt between two objects will be the same magnitude, one force will not be greater than the other when comparing the force of gravity between two objects. Another thing to consider is that if you are just looking for the acceleration due to gravity rather than the force of gravity, we can simplify the equation into this:

$g = G\frac{M}{r^2}$, we don't need to include the mass of one of the objects, which object you may be asking? To find the acceleration due to gravity, we do not include the object with the lesser mass between the two, instead we include the heavier object as the mass used in the equation.

Let's try a problem out:

Example… Niel loves space, and his favorite celestial object is the Moon. He wants to find the force of gravity between his favorite celestial object, the Moon and his home planet Earth. If the mass of the moon is about 7.35×10^{22} kg and the mass of Earth is about 5.98×10^{24} kg. The average distance between the Moon and the Earth is 3.84×10^{8} meters, the mean radius of Earth is 6.37×10^{6} meters, and the mean radius of the Moon is about 1.74×10^{6} meters. Find the force of gravity between both celestial objects.

To solve this equation all we have to do is plug in our values into our equation, one thing to consider is that the distance between both objects must include the distances between both of their radii, this means for the "R" value we must add up all the given values, let's see what that would look like:

$F_g = (6.67\ x\ 10^{-11})\frac{(5.98\ x\ 10^{24}\ kg)(7.35\ x\ 10^{22}\ kg)}{(3.84\ x\ 10^{8}\ m+1.74\ x\ 10^{6}\ m+\ 6.37\ x\ 10^{6}\ m)^2}$. When we solve for the force of gravity, we get a value of about 1.91 x 10^{20} N. The most important thing to consider when doing these types of equations is that you MUST square the distance.

Let's try to solve for the acceleration due to gravity on Earth now!

Mario, who weighs 100 kg, sits on the surface of the Earth. Mario recently forgot the value of the acceleration due to gravity on Earth and wants to determine this value! Mario has only some information, the mass of the Earth is 5.98 x 10^{24} kg, and the mean radius of the Earth is about 6.37 x 10^{6} meters. Help Mario figure out the acceleration due to gravity on the Earth based on the known information.

To solve this question, we will be using our second equation, to determine which mass to use for this equation we have to choose the object with more mass, so in our case the mass of the Earth, not Mario's mass. Now let's plug in these values into our equation:

$g = G\frac{M}{r^2}$, $g = (6.67\ x\ 10^{-11})\frac{(5.98\ x\ 10^{24}\ kg)}{(6.37\ x\ 10^{6}\ m)^2}$. When we solve for the acceleration due to gravity, we find that we get a value of 9.83 m/s^2. We know that the known value of the acceleration on Earth is about 9.81 m/s^2, our value is super close and is something that makes sense! Wow, it's crazy how you can find information like this using simple physics equations! Let's try some example problems:

Example #1: If the gravitational force between two objects is F when they are separated by a distance r, what will be the new force if the distance is halved, and the masses of both objects are doubled? Write your answer in terms of F.

Force: ____________ F

Example #2: Two spheres, each of mass 5 kg, are placed 0.2 m apart. Calculate the gravitational force between them. If the distance between them is doubled, what happens to the gravitational force, does it double, half, etc. Answer with words rather than with a numerical value?

Force: ____________ N

Force After Distance is Doubled: ________________

Example #3: A satellite orbits the Earth at a distance of 1.2×10^7m from the center of the Earth. The mass of the Earth is 5.98×10^{24} kg Calculate the gravitational force acting on the satellite if its mass is 500 kg.

Gravitational Force: ____________ N

Answer Key:

Question #1: To solve this question let's consider what the original equation would look like if all the values are kept as they are: $F_g = G\frac{m_1 m_2}{r^2}$,, now if we were to plug in ½ r for the value of r and plug in $2m_1$ and $2m_2$ let's see what the equation would simply to: $F' = G\frac{2m_1 2m_2}{(\frac{1}{2}r^2)}$, if we were to simplify this we would get

$F' = G\frac{4m_1 m_2}{(1/4)r^2}$, now we can find the ratio between F' and F to see what F' is in terms of F. Let's plug in 5 for both mass variables and 100 for our r value. For both the original equation and the new equation: $F = (6.67\ x\ 10^{-11})\frac{(5\ kg)(5\ kg)}{(100\ m)^2}$, F would equal to 1.6675 x 10^{-13} N, if we plug in the same values into the new equation we get: $F' = (6.67\ x\ 10^{-11})\frac{4(5\ kg)(5\ kg)}{(1/4)(100\ m)^2}$, when we solve for F' we get a value of 1.0672 x 10^{-11} N, if we divide F' by F we get 64. This means the F' is 64 times greater than F which is the original force. There are many ways to solve these types of questions with only variables and no numbers, plugging in numbers is one of the best strategies to easly solve these questions without needing to do some complex and tedious algebra!

Question #2: Let's calculate the gravitational force between both spheres first. All we have to do is plug in the known values into our equation which would look like this: $F_g = (6.67\ x\ 10^{-11})\frac{(5\ kg)(5\ kg)}{(0.2\ m)^2}$, we get a force of gravity of 4.17 x 10^{-8} Newtons. Now if the distance is doubled, we can multiply the 0.2 m by 2 to get a value of 0.4 m, if we plug that back into our equation and solve for the force of gravity we get a value 1.04 x 10^{-8} Newtons. Now if we divide this value by the original force we get 0.25, that means if the distance is doubled, the force is 1/4 th of the original value!

Question #3: This question is pretty simple and just tests our calculator skills, all we have to do is input these values into our equation and solve for the force of gravity, the equation would look like this: $F_g = (6.67\ x\ 10^{-11})\frac{(5.98\ x\ 10^{24}\ kg)(500\ kg)}{(1.2\ x\ 10^{7}\ m)^2}$, if we correctly input these values into our calculator we should get a value of 1384.95 Newtons!

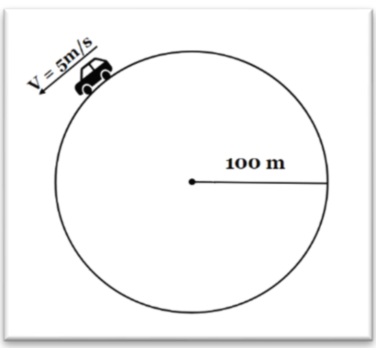
3.15

Dynamics

Centripetal Force

Have you ever turned sharply in a car and felt yourself being pushed against the sides of the car? Well, this phenomenon is known as centripetal force and centripetal acceleration. We know how velocity works for a linear and straight system, but for a system moving in a circular path how would that work? Well, the equation we use for the centripetal acceleration is $a_C = \frac{v^2}{R}$, and for our centripetal force, our equation is very similar, it is $F_C = \frac{mv^2}{R}$. The centripetal force is a net force so the equations can get a bit interesting. Let's try a simple example first:

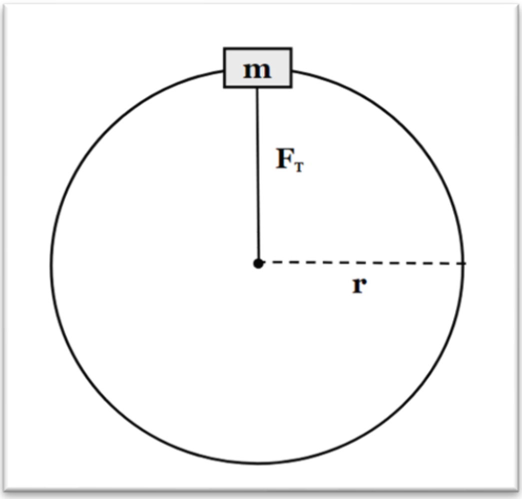
Example... If there is a 100 kg car traveling at a speed of 5 m/s around a circular track which has a radius of 100 meters. Find the centripetal force acting on the car.



You may be wondering why there is acceleration and thus a force if the car is traveling at a constant speed. This is because as you know velocity is a vector which means it's direction **or** magnitude could change for there to be a change in its acceleration. Clearly the direction of the velocity is constantly changing as the car moves around the track which means there is acceleration. But which way would the force and acceleration point towards? Well, if we were to find where there average of all the velocity vectors would point in every point in the circular track, we would see that the **acceleration acts towards the center of the circle**. That means that the force would also act towards the center. One final thing to consider is that the velocity vector is a tangential velocity, this means that if the circular track were to stop suddenly, the car would continue traveling in the same direction as its velocity vector. These are all very interesting things to consider but now since we have this information, we can solve for the centripetal force. Let's plug in our knowns to our equation: $F_C = \frac{mv^2}{R}$. when we plug in our values, we get an equation that looks like this $F_C = \frac{(100\ kg)(5\ m/s)^2}{(100\ m)}$, we get a centripetal force of 25 Newtons!

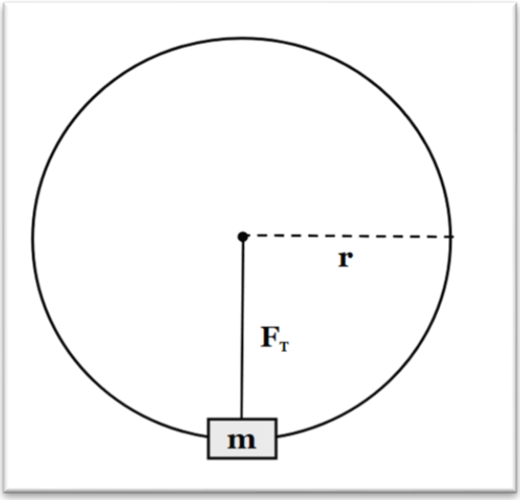
Now that we have learned about horizontal centripetal acceleration and force, we can learn about vertical centripetal acceleration and force. Since we know that centripetal acceleration is a net force, we have to consider if other forces like normal force or the force of gravity are a part of the equation. Let's take a look at the two types of vertical problems:

Example at the Top of a Circle: Let's say a mass is attached to a string and is being spun in a circular path vertically, find the force of tension on the string at the TOP of the circle.



Let's solve this question in terms of variables so we can apply the knowledge to any other question like this. First thing we have to consider is that the centripetal acceleration is a net force, that means that the equation would look like this $\Sigma F_c = ma_c$, so let's add up what the forces would be that act as the net force, since we know the direction of the net force will be towards the center and at this instant that would be downwards, we can consider the net force to be positive in the downwards direction. This would mean that the force of tension and the force of gravity, as they both act downwards, would both be postive. So, our equation at the top will look like this $F_{Top} + mg = ma_c$, from there we can solve for the force of tension at the top.

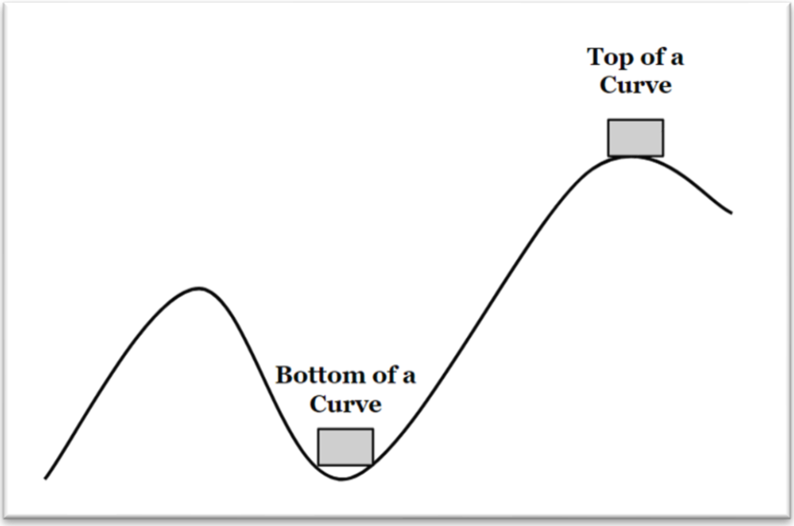
Example at the Bottom of a Circle: Let's say a mass is attached to a string and is being spun in a circular path vertically, find the force of tension on the string at the BOTTOM of the circle.



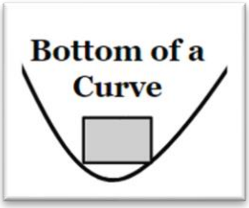
Here we would do a similar setup but now let's consider the direction of the net force. It would be upwards as that's the direction of the center of the circle. This means that the force of tension would be positive as it acts in the same direction as the net force, but the force of gravity would be negative as it acts away from the direction of the net force. So, our equation would look like this $F_{Bottom} - mg = ma_c$.

Now that we have learned how to manipulate vertical circles, we can learn about roller coasters!

Let's talk about the centripetal force at two different instances on a rollercoaster. First at the top of a curve and then at the bottom. Here's what both scenarios would look like:

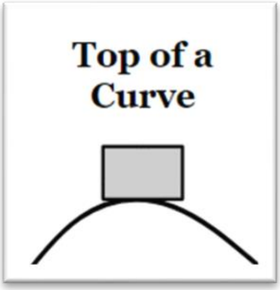


Let's first talk about the scenario at the bottom of a curve!



At the bottom of a curve do you think you would feel heavier or lighter? Well let's find out using our newfound knowledge of centripetal acceleration!

At the bottom of a curve, we know that the centripetal force will be pointing upwards, towards the center of the circle/curve. Knowing that, we also know that there is a normal force as there is a contact point with the curve, and there is a force of gravity. The normal force will be upwards towards the center of the curve while the force of gravity will be negative and downwards as it acts away from the center of the curve. If we were to set up an equation for this scenario, we would get that: $F_N - mg = ma_c$ which would tell us that the normal force would be an addition of the centripetal acceleration force and the force of gravity if we rearrange the equation. This essentially means we would feel heavier as our apparent weight is greater at the bottom of a curve in a roller coaster, cool huh? Well let's learn about what happens at the top of a curve.



Do you ever have that weightless feeling or an emptiness in your stomach at the top of a curve on a roller coaster? Well let's explain the physics behind that!

We know that the net force acts towards the center of the circle, here it will be acting downwards, the normal force will be acting upwards away from the center which will make it negative, and the force of gravity will be acting downwards in the positive direction towards the net force. If we set up the equation, we would get $mg - F_N = ma_c$ which would mean that the normal force would be equal to the force of gravity minus the centripetal force. This would mean that the normal force is less than the force of gravity giving us that weightless feeling! Centripetal acceleration is really cool and can be applied to many things! Let's complete some practice problems now that we have learned all this new information!

Example #1: A racecar driver drives around a circular track at 20 m/s. The track's radius is 200 meters and the driver, and his car weigh a total of 500 kg. Find the centripetal force acting on the driver and his car.

Centripetal Force: _______________ N

Example #2: Brianna spins a 5 kg hammer around in a vertical circle using a string at a constant velocity of 3 m/s. The length of the string is 5 meters. Find the force of tension on the string at the top of the circular path.

Force of Tension at The Top: _______________ N

Example #3: Paul is on a rollercoaster going at a constant speed of 10 m/s. Paul and rollercoaster weigh a total 1000 kg, find the apparent weight of Paul and the cart at the bottom of a circular loop with a radius of 3 meters.

Apparent Weight: ________________ N

Answer Key:

Question #1: For this question we will simply just plug in the given values into our equation for the centripetal force which is: $\sum F_C = \frac{mv^2}{R}$. When we plug in the values given to us, we get an equation that looks like this: $\sum F_C = \frac{(500\,kg)(20\,m/s)^2}{(200\,m)}$ when we solve for the force, we get a value of 1000 Newtons!

Question #2: For this question we can use our equation that we derived before for the force of tension at the top of a vertical circular loop. Our equation is $F_{Top} + mg = ma_c$ but if we manipulate it to find only the force of tension at the top, we get an equation that looks something like this: $F_{Top} = ma_c - mg$when we plug in our given values we get this:

$F_{Top} = \frac{(5\,kg)(3\,m/s)^2}{(5\,m)} - (5\,kg)(-10\,m/s^2)$, if we solve for the force value we get a value of 59 Newtons. Remember to make sure that the value of the acceleration due to gravity is a negative value here since direction matters.

Question #3: This question is similar to question 2, let's take our equation for roller coasters which involves the cart at the bottom of a circular loop to solve for the apparent weight, better known as the normal force. Our equation is $F_N - mg = ma_c$, when we move around the variables to solve for the normal force we get an equation that looks like this: $F_N = mg + ma_c$, let's plug in our knowns and solve for the normal force. Our equation with all our knowns plugged in will look like this:

$F_N = \frac{(1000\ kg)(10\ m/s)^2}{(3\ m)} + (1000\ kg)(-10\ m/s^2)$, when we solve for the normal force, we get a value of 23,333.33N. The person would feel significantly heavier during this part of the ride!

Chapter 4

Energy & Momentum

4.1

Energy & Momentum

Momentum: Mass in Motion

Momentum is considered to be the "*quantity of motion*", but what is it? And what are its applications in physics? Momentum is a common value seen across physics, and thus it was given a name and present in many equations.

Momentum is expressed as:

$$p = mv$$

Where m is mass and v is velocity.

This means that a heavy object with a non-zero velocity has more momentum than a lighter object with the same velocity, and an object with a greater velocity has more momentum than an identical object with less velocity.

We can actually express Newton's Second Law in terms of momentum. This has uses as it allows us to see the relationship between force and change in momentum.

With change in momentum being p = mv:

$$F = ma$$

$$F = m\frac{\Delta v}{\Delta t}$$

$$F = \frac{\Delta p}{\Delta t}$$

We can see through this equation that if a force is being applied on a system for any amount of time, it will incur a change in the object's momentum. If a small force is applied for a long period of time, it will result in the same change in momentum as a large force being applied for a short period of time. This change in momentum "p" is also known as impulse. The units for momentum are kg·m/s which can also be written as N·s. Let's try some examples with our new knowledge.

Example #1: A soccer ball that weighs 5 kg is moving at a constant velocity of 4 m/s. What is its momentum?

Momentum: _______________ kg·m/s

Example #2: A car gets hit by another car from the back with a force of 1000 N over a period of 0.1 seconds. What is the change in momentum of the car, assuming the surface is frictionless?

Change in Momentum: _______________ N·s

Example #3: A 150 kg sailboat going at a constant velocity of 6 m/s encounters a humongous wave with a force of 1200 N. After 3 seconds, the wave passes. What is the sailboat's new momentum?

Momentum: _______________ kg·m/s

Answer Key:

Example #1: Since we know the mass and velocity of the soccer ball, all we need to do to find the momentum of the ball is to plug into the momentum equation $p = mv$. Doing this, we find

$$p = (5\,kg)(4\,ms) = 20 \text{ kg} \cdot \text{m/s}$$

Example #2: Using the equation $F = \frac{\Delta p}{\Delta t}$, we can plug in our values for F and t to solve for p, the change in momentum. We find that $\Delta p = F\Delta t$ by multiplying both sides by t. Plug in 1000 N for F and 0.1 s for t.

$$p = (1000\,N)(0.1\,s) = 1000 \text{ N} \cdot \text{s}$$

Example #3: The question asks for the final momentum of the sailboat, meaning we need to first find the initial momentum and the impulse. To find the initial momentum, we can plug in 150 kg for the mass of the sailboat and 6 m/s for the velocity in $p = mv$. We find $p = (150\,kg)(6\,m/s) = 900\ \mathrm{kg \cdot m}/s$ for the initial momentum. Next, we can use the equation $\Delta p = F\Delta t$ to find the impulse:

$\Delta p = (-1200\,N)(3\,s) = -3600\mathrm{N \cdot s}$ (The force of the wave is negative as it is opposite in direction to the sailboat's motion). Finally, we need to find the sum of the initial momentum and impulse.

$$\begin{aligned} p_f = p_i + \Delta p &= (900\ \mathrm{kg \cdot m/s}) + (-3600\ \mathrm{N \cdot s}) \\ &= -2700\ \mathrm{kg \cdot m/s} \end{aligned}$$

4.2

Energy & Momentum

Work, Energy, and Power

When we think of work, we often think about careers, but what is it in physics? Work is defined as energy transfer as a result of an external force being applied to a system over a displacement. Let's take a look at the equation:

$$W = Fdcos(\theta)$$

Where W is work, F is the magnitude of the external force, d is the magnitude of the displacement, and θ is the angle between the force and displacement vectors.

We know what the definition of work is, but what does it mean in practical terms? If we push a heavy object across the surface of the floor, then work is being done on that object; the force we apply is causing the object to be displaced. Now, imagine holding that heavy object in your hands and you start walking. Although the force you are applying is vertical to counteract the force of gravity, the object is being displaced in the horizontal direction. Since the force vector and displacement vectors are perpendicular, then the work being done by you is equal to 0 as $cos(90°) = 0$.

Looking back at the definition of work, we see that it is also a transfer of energy. If you rub your hands against each other, you can actually feel this transfer of energy.

Your hands are doing work on one another through friction and are being displaced, and this is transferred through heat energy, which is why your hands feel warmer after rubbing them together. How can we express this transfer of energy using math?

We can find net work by substituting net force for F. Doing so gives us:

$W_{net} = F_{net}d$ (we are able to remove $cos(\theta)$as net force will always be parallel to the displacement, thus the angle between the force and displacement vectors is 0, and cos(0°) = 1.

Using this, we can substitute F_{net} for $m\ x\ a$ as stated by Newton's Second Law.

$$W_{net} = m\ x\ a\ x\ d$$

Finally, using the kinematic equation $v_f{}^2 = v_i{}^2 + 2ad$, we get the following:

$$W_{net} = m(\frac{v_f{}^2 - v_i^2}{2d})d$$

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

This equation is the work-energy theorem, and states that net work equals a change in the quantity $\frac{1}{2}mv_f^2$. But what does this have to do with energy?

$\frac{1}{2}mv_f^2$ is actually a quantity we call linear **kinetic energy** (KE), which is the energy of an object with mass which is in motion. But kinetic energy is only one facet of what we call energy. All energy is either potential or kinetic, and the total energy within a system is called mechanical energy.

Kinetic energy, as stated before, has to do with motion. Any object that has mass and moves has kinetic energy. For example, heat energy is kinetic as it is simply the vibration or movement of particles.

Potential energy ($PE\ or\ U$) is energy that is stored. For example, a rock on top of a hill has gravitational potential energy as the height of the rock gives it the capacity to roll down. We will explore types of potential energy in future lessons.

Mechanical energy is the sum of kinetic and potential energy ($ME = KE + PE$), and always remains constant unless an external force is being applied on the system. Mechanical energy always changes when work is done on a system.

Work and energy also possess the same units, joules (J), which are equal to Newton-seconds (Ns) or kilogram-meters per second square (kgm/s^2).

Finally, relevant to work and energy is power. When we think of power, laser beams or explosions often come to mind. But in physics, power is defined as the rate at which work is done.

$$P = \frac{W}{t}$$

The units for power are watts (W), where one watt is equal to one joule per second.

Let’s try some example problems:

Example #1: Find the work done by a person applying a force of 30 N on a box 60° above the horizontal over a displacement of 5 meters.

Work: ______________ J

Example #2: A 50 kg person is ice skating at a constant velocity of 4 m/s to the right when their friend pushes them to the right with a force of 100 N over 6 meters. What is the person's new velocity?

Velocity: ______________ m/s

Example #3: Computers use a power supply unit to power each part. A power supply unit boasts a wattage of 750 W. How much work is done by this power supply unit over a time period of 20 seconds?

Work Done Over Time: _____________ Watts

Answer Key:

Example #1: Looking at the work equation $W = Fdcos(\theta)$, we see that we have all the values necessary to solve the work. This gives us

$$W = (30\,N)(5\,m)cos(60°) = 75\,J.$$

Example #2: Since we need to find the person's final velocity after the push, we need to find the final kinetic energy. We can do this by taking the initial kinetic energy and adding the work done during the push. For our initial kinetic energy, we get

$KE_{initial} = \frac{1}{2}(50\,kg)(4\,m/s)^2 = 400\,J$. Now, we need to find the work (the change in energy due to the work-energy theorem): $W = (100\,N)(6\,m)cos(0°) = 600\,J$. Finally, we add the work to the initial kinetic energy to get the final KE: $KE_{final} = 400\,J + 600\,J = 1000\,J$. Using this, we can take the kinetic energy formula and solve for our velocity.

$$KE_{final} = \frac{1}{2}m(v_{final})^2$$

$$\frac{2KE_{final}}{m} = (v_{final})^2$$

$$\sqrt{\frac{2KE_{final}}{m}} = v_{final}$$

Plugging in our known values in this equation, we get approximately 6.3 m/s for v_{final}.

Example #3: We know $P = \frac{W}{t}$, so if we multiply both sides by t we get $Pt = W$. We know power and the time, so we just plug it in to find the work:

$$(750\,W)(20\,s) = 1500\,J = W.$$

4.3

Energy & Momentum

Gravitational Potential Energy

One of the most commonly seen examples of potential energy is gravitational potential energy. Gravitational potential energy represents a separation of two objects which are attracted gravitationally. Knowing that work represents a change in energy, we can use this to find the gravitational potential energy. Since the object is at equilibrium, then work done must have a force equal in magnitude but opposite in direction to gravity.

$W = \Delta PE = Fd\,cos\theta = 1$ (in this case as the force and distance vectors have an angle of 0° between them)

$$\Delta PE_g = -F_g d$$

$\Delta PE_g = mg\Delta h$ (h represents the distance between the two objects)

Also, we can solve for potential energy using the following force equation:

$F_g = G\frac{m_1 m_2}{r^2}$. However, since we are not using the Earth, and potential energy is relative, we need to set our "zero point" where the potential energy is equal to zero. On Earth, we set this zero point to the ground, where PE increases as you move further away from the ground. Seeing how work is needed to separate masses as they are attracted by gravity, then a distance of ∞

between the two objects is our maximum possible gravitational potential energy. Because of this, it is convenient for us to set the zero point as the maximum potential energy where the distance r between the two objects equals ∞. As 0 J is the maximum possible potential energy, anything lower than that must be negative, which explains why we include the negative sign in the final equation.

$$\Delta PE_g = -F_g d$$

$\Delta PE_g = -G\frac{m_1 m_2}{r^2}$ (Since d and r represent the same value, we can substitute r for d)

$$\Delta PE_g = -G\frac{m_1 m_2}{r}$$

Both equations for gravitational potential energy are valid depending on the reference frame chosen. Let's solve some practice problems to understand this new concept:

Example #1: Find the gravitational potential energy of a 0.5 kg toy ball at a height of 20 m.

PE: ________________ J

Example #2: A 2000 kg satellite orbits the Earth at a distance of 2×10^6 m away from the center of the Earth. Given the gravitational constant $G = 6.7 \times 10^{-11}$ and the mass of the earth is 6×10^{24} kg, find the gravitational potential energy of the satellite.

PE: ________________ J

Example #3: Jimmy throws a 0.1 kg coin into a 5 m deep wishing well for good luck. What is the velocity of the coin right as it is about to hit the bottom of the well? (Use 10 m/s^2 for gravitational acceleration.)

Velocity: _________________ m/s

Answer Key:

Example #1: We simply plug in the known values into the formula $PE_g = mgh$. This gives us a final answer of $PE_g = (0.5\ kg)(10\ m/s^2)(20\ m) = 100\ J$.

Example #2: Since the satellite and Earth are separated by a large distance, it is likely that the gravitational field strength "g" measured on Earth's surface will give us an inaccurate answer. Thus, we need to use the formula $\Delta PE_g = -G\frac{m_1 m_2}{r^2}$ to find the potential energy. We need to plug our values into the equation. $-(6.7\ x\ 10^{-11})\frac{(2000\ kg)(6\ x\ 10^{24}\ kg)}{(2\ x\ 10^6\ m)^2} = -4.02\ x\ 10^{11}\ J$.

Example #3: We know potential energy can be converted to kinetic energy, and that mechanical energy is conserved if no work is done to put in or take out energy from the system. In this case, we know that the mechanical energy is conserved in our coin-Earth system. For most convenience, we can set the bottom of the well to our zero point where $h = 0$. As the coin approaches the zero point, its potential energy will be converted (h gets smaller and smaller, v gets bigger and bigger) to kinetic energy. This means that the instant before the coin touches the bottom, the potential energy at the beginning will equal the kinetic energy at the end.

$$mgh_{initial} = \frac{1}{2}m(v_{initial})^2$$

$$2gh = (v_{final})^2$$

$$\sqrt{2\mathrm{gh}} = v_{final}$$

Plugging in our known values, we get a final velocity of 10 m/s.

4.4

Energy & Momentum

Elastic Energy

Ever notice how when you compress or pull a spring really hard, it starts to oscillate back and forth? This is an example of something called **elastic energy**. Elastic energy represents the energy stored within an object after work is applied and is a type of potential energy. When you stretch a rubber band or pull a spring, you are doing work onto the object causing an elastic deformation and elastic energy to be stored in this object.

Elastic energy (PE_s) is given by the equation:

$$PE_s = \frac{1}{2}k\Delta x^2$$

Where k is the spring constant and x the change in position from the object's equilibrium. Equilibrium is a state where no net work is being done to the object, and thus the object experiences no deformation. In a spring, this would look like the spring's original position without being pushed or pulled.

Elasticity means that the object is able to go back to its original state, but what happens when you apply so much force that the object can't revert? In this case, the object is incapable of storing all the energy which was transferred to it in the form of elastic energy. Let's do some practice problems based on this topic.

Example #1: A spring has a constant k = 0.5 N/m and is stretched 0.2 m. What is the elastic potential energy of the spring?

PE_s: ________________________ J

Example #2: A 5 kg block slides on a frictionless surface with a constant velocity of 4 m/s. The block then comes into contact with a spring with constant k of 6N/m. What distance is the spring stretched?

Δx: __________________________ m

Example #3: What is the work done to a spring with constant k of 32 N/m after being stretched 4 meters?

Net Work: ______________________________ J

Answer Key:

Example #1: We simply need to plug into the formula $PE_s = \frac{1}{2}k\Delta x^2$ with our known values. We find $PE_s = \frac{1}{2}\left(0.5\,\frac{N}{m}\right)(0.2\,m)^2 = 0.01\,J$.

Example #2: We can convert the kinetic energy of the block to the elastic potential energy of the spring, as work is being done by the block to transfer energy into the spring.

$$KE_{block} = PE_s$$

$$\frac{1}{2}m_{block}v_{block} = \frac{1}{2}k\Delta x^2$$

$$\frac{m_{block}v_{block}}{k} = \Delta x^2$$

$$\sqrt{\frac{m_{block}v_{block}}{k}} = \Delta x$$

Plugging in our known values for the mass of the block, velocity of the block and the spring constant, we find that the spring is stretched by approximately 1.8 meters.

Example #3: We know that the spring force is $F_s = -kx$, and work is $W = Fdcos(\theta)$. We can plug the spring force into the equation to find that $W = k\Delta x^2 cos(\theta)$ (d can be substituted by Δx as they are the same value). This gives us $W = (32\ N/m)(4\ m)^2 cos(180°) = -512\ J$. The reason θ is 180° because the force and displacement vectors are opposite in direction; as you pull the spring, the force is attempting to compress it back to its original state.

4.5

Energy & Momentum

Law of Conservation

Both momentum and energy are conserved values. But what does this mean?

Momentum is conserved on any system should the external net force be equal to 0. This means that even if the momentum of the parts in the system change, the momentum of the entire system remains the same if no external force is applied to it. If a ball is thrown and hits a stationary block, then the momentum of each object changes, but the momentum of the ball-block system remains the same. This principle applies no matter how many objects are in this system.

We can use the mathematical definition of impulse to visualize this principle:

$$\Delta p = F\Delta t$$

$$\Delta p = 0N \; x \; \Delta t$$

$$\Delta p = 0 \; N \; x \; s$$

Since the change in momentum is equal to zero, then momentum must be constant.

We can also represent this principle with the following:

$$p_{total,initial} = p_{total,final}$$

Where $p_{total,initial} = p_{1,initial} + p_{2,initial} + \cdots$ *and*

$$p_{total,final} = p_{1,final} + p_{2,final} + \ldots$$

Momentum is also independent based on direction, so horizontal momentum may be conserved despite a changing vertical momentum.

Similarly, energy follows a principle of conservation. Mechanical energy (the sum or total energy in a system if you recall) remains constant unless work is done on that system.

$$ME_{initial} = ME_{final}$$

For example, let's take the example of a ball dropping from rest on Earth and set our system to the ball-Earth system. Even though the ball has its maximum gravitational potential energy at the very start of motion where its position is the greatest and the maximum kinetic energy at the bottom where its velocity is the greatest, the total mechanical energy will remain constant as no energy from outside the system was transferred in.

Additionally, the **law of conservation of energy** states that energy cannot be created nor destroyed, rather it can only be transferred from one system to another or transformed into another form. Everywhere around us, we experience this transformation and transference of energy. When water turns a water wheel, the kinetic energy is transformed into electrical energy. When an object hits a spring, the kinetic energy of the object transfers to a system of the object and the spring, also transforming into elastic energy. Let's try some practice problems with this new concept!

Example #1: A 2 kg block slides across a frictionless surface at a constant velocity of 5 m/s. Eventually, the block comes into contact with a stationary 5 kg block and sticks to it. What is the new velocity of the 2 kg block?

Velocity: ________________ m/s

Example #2: A 3 kg block slides across a frictionless surface at a constant velocity of 6 m/s. It comes into contact with a stationary 10 kg block. After the collision, the 10 kg block has a constant velocity of 10 m/s. What is the new velocity of the 3 kg block?

Velocity: ________________ m/s

Example #3: A 0.7 kg ball falls from rest at a height of 10 meters. What is the final mechanical energy of the ball-Earth system?

Mechanical Energy: _____________ J

Answer Key:

Example #1: We know that momentum is conserved in this situation as there is no net force acting on the two block system. Because of this, we can set up our conservation of momentum equation.

$$p_{initial} = p_{final}$$

$$p_{1,initial} + p_{2,initial} = p_{final}$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_{final}$$

Since the two blocks stick after they come into contact, we need to add their masses together for the final momentum.

$$(2\ kg)(5\ m/s) + (5\ kg)(0\ m/s) = (2\ kg + 5\ kg) v_{final}$$

$$10\ kgm/s = (7\ kg)v_{final}$$

$$1.43\ m/s \cong\ v_{final}$$

The question asks for the velocity of the 2 kg block, but technically the answer we got was the velocity of the entire system. However, since the 2 kg block is stuck with the 5 kg block, then the velocity of the system is the same as the velocity of the block.

Example #2: Similarly to the last problem, there is no net force acting upon the system, so we can set up a conservation of momentum equation.

$$p_{initial} = p_{final}$$

$$p_{1,initial} + p_{2,initial} = p_{1,final} + p_{2,final}$$

$$m_1 v_{1,initial} + m_2 v_{2,initial} = m_1 v_{1,final} + m_2 v_{2,final}$$

Now, we just plug in our values:

$$(3\ kg)(6\ m/s)\ + (10\ kg)(0\ m/s) = (3\ kg)v_{1,final} + (10\ kg)(10\ m/s)$$

$$18\ kgm/s + 0\ kgm/s = (3\ kg\ )v_{1,final} + 100\ kgm/s$$

$$-82\ kgm/s = (3\ kg)v_{1,final}$$

$$-27.3\ m/s\ \cong v_{1,final}$$

We see that in this case, the velocity is negative. This means that in order to conserve the momentum in this system, the 3 kg block changes direction.

Example #3: In this scenario, no net work is being done to the ball-Earth system. This means that the mechanical energy will be conserved and the change in mechanical energy throughout the motion will be equal to 0. Thus, we can simply find initial mechanical energy. At the very top of the motion, the ball is at rest, meaning that $KE = 0$. We know $ME = PE + KE$, so if $KE = 0, ME = PE$. By finding the initial gravitational potential energy, we end up finding the mechanical energy of the system.

$$ME = PE_{g,initial}$$

$$ME = mgh$$

$$ME = (0.7\ kg)(10m/s^2)(10\ m)$$

$$ME = 70\ J$$

Our final mechanical energy is 70 J.

4.6

Energy & Momentum

Elastic & Inelastic Collisions

In physics, we consider two different types of collisions, **elastic** and **inelastic**. A collision is defined as a sudden contact between two objects. How do these two collisions differ?

An elastic collision means that no kinetic energy is lost within a system. When two objects collide elastically, then the sum of their initial kinetic energies is equal to the sum of their final kinetic energies. True elastic collisions only happen on an extremely small scale and at ideal circumstances.

Inelastic collisions are any collision that is not considered elastic. In an inelastic collision, kinetic energy is lost due to internal forces, causing this energy to be transferred elsewhere. A perfectly inelastic collision is two objects sticking together after colliding, as it loses the most kinetic energy.

Both collisions observe conservation of momentum. As long as no external force is being applied to the system, then no matter how the objects interact, the momentum of the system will be conserved. For example, if two objects collide and stick together, then the sum of the momentum of each object will be equal to the momentum of the two objects stuck together.

4.7

Energy & Momentum

Conservative & Non-Conservative Forces

When looking at energy, forces are split up into two categories: **conservative** and **non-conservative**.

Work is defined by the equation $W = Fdcos(\theta)$, where d is displacement. As we learned in the kinematics chapter, displacement doesn't necessarily mean distance. So, is there any change in work if we consider the distance the object travels?

This is the difference between conservative and non-conservative forces. A conservative force means that the work done by the force does not change depending on the path taken by the object. This means that if we consider the distance the object travels, the work done by a conservative force should be the same. An example of a conservative force is gravity. No matter how the object moves, the work done by gravity will always be the same. This is because work done by gravity only depends on the final change of height.

A non-conservative force **does** depend on the path taken. For example, friction is a non-conservative force as the more distance an object travels in between the start and end points, the more time friction is being applied compared to the displacement of an object.

Another way to look at conservative and non-conservative forces is that conservative forces only depend on the start and end point of the object while non-conservative forces rely on the entire motion.

Chapter 5

Rotational Motion

5.1

Rotational Motion

Radians

You might've seen **radians** (symbol rad) being used as a substitute for degrees. They are the SI unit for angle measurement and are commonly used in physics. But where do they come from? A radian is defined as the angle where the arc length of a circle is equal to the radius. In total, a circle has 2π radians.

$$\theta = \frac{s}{r}$$

Where θ is the angle measure, s is the arc length, and r is the radius. When s and r are equal, $\theta = 1$. Thus, our definition of the radian.

Since there are 2π radians and 360 degrees in a circle, we can use this to convert between the different angle measurements.

$$\theta_{radians}\ x\ \frac{180}{\pi} = \theta_{degrees}$$

For reference, 1 rad is approximately 57.3 degrees.

5.2

Rotational Motion

Angular Displacement, Angular Velocity, and Angular Acceleration

In this book, we've talked about linear and projectile motion, but the scope of motion far exceeds that. In this chapter, we'll explore **rotational motion**, which is the motion of an object traveling in a circular path. Luckily, rotational motion follows a lot of the same principles that we've seen previously in kinematics.

As with any physics problem, we need to define our reference point and axis of motion. We've seen right as positive and left as negative in linear motion... but this time we aren't moving linearly. So, what should we do? Instead, we can define our axes by clockwise and counterclockwise rotation. For our reference point, we need to define where the angle measure equals 0.

Imagine a Ferris wheel that is just starting up and rotating clockwise. Let's set the very bottom of the wheel to $\theta = 0\ rad$ and clockwise motion to positive. If the Ferris wheel rotates clockwise to an angle of 2 rad, then we say its angular displacement is equal to 2 rad. Angular displacement, similar to linear displacement, is a vector and the difference between the final and initial angular positions.

Using the same example of the Ferris wheel, let's say that after the initial startup, it rotated 2 radians in 10 seconds. If we find the change in angular position per second (which is 2 radians divided by 10 seconds), we get a quantity known as angular velocity. The angular velocity is also a vector and is the rate of angular displacement. We find that the angular velocity of the Ferris wheel is 0.2 radians per second (rad/s), meaning that every second, the Ferris wheel will rotate 0.2 radians.

We said that the Ferris wheel was just starting up from rest, and now rotates at an angular velocity of 0.2 rad/s. Now, let's say that it took the Ferris wheel 5 seconds to fully start. If we see how the angular velocity changes in this time, we find a quantity known as angular acceleration. Angular acceleration, like the previous quantities, is a vector and describes how the angular velocity changes per second. In this example, we know the angular velocity changes from 0 to 0.2, and took 5 seconds to get to that point, meaning that the angular acceleration throughout that time was 0.04 rad/s^2.

5.3

Rotational Motion

Rotational Kinematic Equations

There is obviously a parallel between linear and rotational kinematics described in the previous chapter, but how does this reflect in the rotational kinematic equations?

The rotational kinematic equations are:

1. $\theta_f = \theta_i + \omega t + \frac{1}{2}\alpha t^2$
3. $\omega_f = \omega_i + \alpha t$
4. $\omega_f^2 = \omega_i^2 + 2a\Delta\theta$

Where θ is angular position, ω is angular velocity, and a is angular acceleration.

The first equation solves for the final angular position given initial angular position, angular velocity, angular acceleration and time. The second equation solves for final angular velocity given

We can see that the parallel goes past just conceptually, but mathematically too. The rotational kinematic equations are simply the linear kinematic equations with the angular quantities replacing the linear quantities. We can manipulate these equations to give us the desired quantities. When faced with a physics problem, it is best to make your selection of which equation to use after listing all your known quantities.

Since the equations and quantities are nearly identical, then analyzing motion graphs in rotational kinematics is nearly the same as linear kinematics, as well! We previously said that the slope of a position vs time graph is equal to the velocity of the moving object, and the slope of the velocity vs time graph is equal to the acceleration. Turns out, this also applies for angular quantities! Let's try some example problems:

Example #1: A record is placed into a record player at rest and after 0.5 seconds rotates at a constant angular velocity of 0.7 rad/s. What is the angular acceleration of the record player from when the record is placed to when it starts to rotate at a constant velocity?

α: ____________________ rad/s^2

Example #2: A person gets on a bicycle which is at rest. After the bicycle wheels have made 135 revolutions, the angular velocity of the wheel is 10 rad/s. What is the angular acceleration of the bicycle wheels?

α: ____________________ rad/s²

Example #3: A centrifuge starts at rest and has a constant acceleration of 100 rad/s². After 45 seconds, how many revolutions has the centrifuge made?

α: ____________________ rad/s²

Answer Key:

Example #1: We are given time, final angular velocity, and initial angular velocity and need to find the angular acceleration. Based off this information, the equation with the best fit is $\omega_f = \omega_i + \alpha t$. We can simply plug in our known values into the equation to solve for the angular acceleration.

$$\omega_f = \omega_i + \alpha t$$
$$(0.7\, rad/s) = (0\, rad/s) + \alpha(0.5\, s)$$

When we solve for the rotational acceleration, we find that our answer is 1.4 rad/s^2.

Example #2: Looking at the problem, we see that we are given the initial angular velocity, final angular velocity, and angular displacement. All we need to find is the angular acceleration, and based off the example the best fitting equation is $\omega_f^2 = \omega_i^2 + 2a\Delta\theta$. Let's plug in our values:

$$\omega_f^2 = \omega_i^2 + 2a\Delta\theta$$
$$(10\, rad/s)^2 = (0\, rad/s)^2 + 2a(848.23\, rad)$$

When we solve for our angular acceleration, we get a value of 0.059 rad/s^2.

Example #3: In this example, we're given an angular acceleration of 100 rad/s^2 and a time of 45 seconds, all while trying to find the angular displacement. Since the centrifuge starts from rest, we can say that the initial angular velocity is equal to 0 rad/s. The equation of best fit would be $\theta_f = \theta_i + \omega t + \frac{1}{2}\alpha t^2$ as it has all the necessary values we need to find our answer.

$$\theta_f = \theta_i + \omega t + \frac{1}{2}\alpha t^2$$

$$\theta_f = (0\,rad) + (0\,rad/s)(45s) + \frac{1}{2}(100\,rad/s^2)(45s)^2$$

When we solve for the θ_f, we find it is equal to 101250 rad.

However, this isn't our final answer. The question is asking for how many revolutions have occurred after 45 seconds, not the angular displacement in radians. A revolution is the full turn of a spinning object. We know that a circle has 2π radians, so we need to divide the value we got by 2π. This gives us a value of 16114.43... However, because we're looking at full revolutions, we can neglect everything after the decimal point—giving us the final answer of 16114 revolutions!

5.4

Rotational Motion

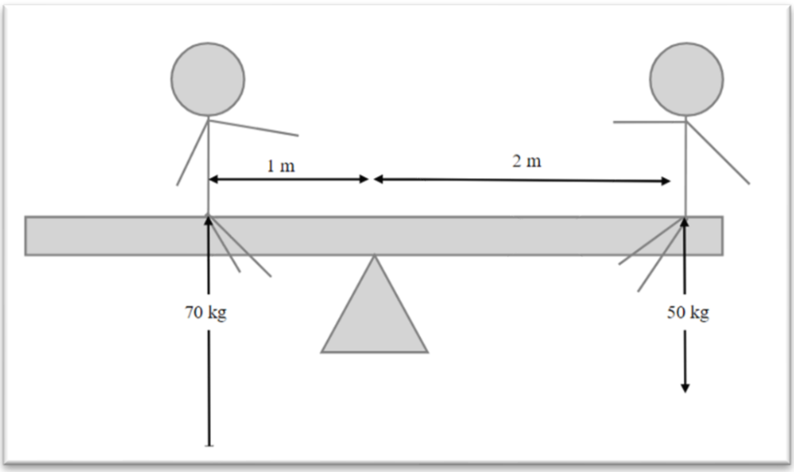
Torque

Remember when we learned about forces? Well, the force equivalent for the rotational motion unit is known as **torque**. Torque is defined as the measure of a force which causes a rotation. If a force causes a change in the motion of a linear object, then torque is a force which causes a motion of a rotational body. The equation of torque is equal to the force applied on an object multiplied by the distance the force is applied from the pivot point. The pivot point is where the object will rotate, unless specifically mentioned by the question the pivot point will be the center of gravity of an object. The equation of torque looks like this: $\tau = rFsin\Theta$

The sine theta is extremely important as if a force is applied at an angle, the torque applied will be less than if the force was acting straight on the object. Also, the larger the distance from the pivot point (which is the "r" value known as the radius) the greater the torque will be. The units for torque are Nm (newton-meters). Let's take a look at an example:

Example… Sean sits on a seesaw 2 meters right from the pivot point. Sean weighs 50 kg and applies a torque directly downwards on the seesaw. Cael sits on the other side of the seesaw 1 meter from the pivot point. He weighs 70 kg and applies a torque directly downwards on the seesaw. Find the direction of the torque and its magnitude.

To begin answering this question we should first draw a diagram of what is happening:



Now that we have a diagram, we must learn one more concept. If a torque is applied which causes a system to rotate **clockwise**, the torque is considered negative. If a torque is applied which causes a system to rotate **counterclockwise**, the torque is considered positive. From our example, Cael will cause the seesaw system to rotate counterclockwise while Sean will cause the seesaw system to rotate clockwise. The force applied is just the weight of each person so we will simply find their weight by multiplying their mass with the acceleration due to gravity on Earth. Now that we know this information, let's use our equation to solve for the total torque of the system. One important piece of information to consider is that if a force is acting perpendicular to the surface such as in this scenario, the angle for the sine theta value will be equal to $sin(90°)$ which is just equal to 1. Let's first find the torque exerted by Sean, if we plug in the information, we know into our equation remembering that the value will be negative as the torque is clockwise, we get an equation that looks like this: $\tau = -(2\,m)(500\,N)sin(90°)$. When we solve this to find the torque, we get a value which is -1000 Nm. Now let's find the torque of Cael, $\tau = (1\,m)(700\,N)sin(90°)$ which is equal to 700 Nm. Now let's add both torques together, $-1000\,Nm + 700\,Nm$ gives us -300 Nm. This means that the torque is equal to 300 Nm in the negative direction which is the clockwise direction.

What would the torque of Cael's force be if it was acting at an angle of 70°. Let's try to answer this question. Essentially all we will be changing in the equation for Cael is the sine theta value. Let's plug in 70° for the angle. $\tau = (1\,m)(700\,N)sin(70°)$, when we solve the equation for the value of the torque, we see that it is equal to 657.78 Nm which is less than the 700 Nm we had when using 90°. This means that the **maximum torque** is when the angle is equal to 90° and anything more or less than this angle will mean there is less torque than the maximum possible when using 90°.

5.5

Rotational Motion

Newton's Second Law for Rotational Motion

We understand that torque is simply just force about an axis. Previously, we also went over Newton's Laws, one of the most important contributions to our understanding of forces. Do these laws relate to torque?

Truth is, they definitely do. Throughout this chapter, we have seen rotational motion act as a parallel to linear motion with nearly identical equations and concepts. This same parallel also applies to **Newton's 2nd Law**. If you recall, Newton's 2nd Law states:

$$F_{net} = m\ x\ a$$

Where F_{net} is the net force, m is the mass, and a is the acceleration.

Since force and torque are parallels, this equation also applies to torque, just with a little change. The new equation is:

$$\tau_{net} = I\ x\ \alpha$$

Where τ_{net} is net torque, I is rotational inertia, and α is angular acceleration.

Wait a minute… What's rotational inertia? Why is it taking the place of mass? As we understand it, mass is a measure of the inertia of an object—how “lazy” it is. Meanwhile, mass isn’t enough when describing rotational motion as two wheels could have the same mass but look and act differently. For example, a ring and a coin might have the same mass, but because the ring is hollow in the middle and the coin is not, this causes a difference in how they roll down.

In essence, rotational inertia is a measure of how “lazy” an object is, just while rotating about an axis. Rotational inertia can be calculated, but only in certain situations. If an object is a “point mass”, a mass which can be represented by a point (like a planet in orbit, for example), then its rotational inertia can be calculated by $I = mr^2$. The units for rotational inertia are kg·m^2. Let’s try some practice problems!

Example #1: A 30 kg kid sits on a massless seesaw which is at rest. The kid sits 1 meter from the center of the seesaw. What is the kid’s angular acceleration?

Angular Acceleration: ________________ rad/s^2

Example #2: A person turns a bottle cap with rotational inertia of 0.5 kg·m² at angular acceleration of 0.3 rad/s². What is the torque the person is applying?

Torque: ________________ Nm

Example #3: A water wheel with rotational inertia of 5000 kg·m² is newly installed and starts from rest. After 30 seconds of being in contact with the water, the wheel has made 12 revolutions. What is the torque being applied by the water?

Torque: ________________ Nm

Answer Key:

Example #1: In order to solve this problem, we need to make use of Newton's Second Law and the mathematical definitions of torque and rotational inertia. We know that $\tau_{net} = I\ x\ \alpha$, and that $\tau = rFsin(\theta)$ and $I\ =\ mr^2$, so we can plug these to the original equation.

$$rFsin(\theta) =\ mr^2\alpha$$

$$Fsin(\theta) =\ mr\alpha$$

Since the child is sitting down on the seesaw, F must be the weight of the child, $sin(\theta) =$ can be removed as the force of gravity is perpendicular to the radius of the seesaw, meaning the angle is 90 degrees $sin(90°)\ =\ 1$. Force of gravity is given by $F_g = mg$, so let's plug in mg for the value F.

$$mg =\ mr\alpha$$

$$g =\ r\alpha$$

$$10m/s^2 = (1\ m)\alpha$$

If we solve for the angular acceleration, we can see that our answer is that the kid has an angular acceleration of 10 rad/s^2.

Example #2: For this example, we simply just plug in our rotational inertia and angular acceleration into Newton's Second Law.

$\tau_{net} = I \, x \, \alpha$

$$\tau_{net} = (0.5\, kg \cdot m^2)(0.3\, rad/s^2)$$

When we solve for the torque, our answer is 0.15 Nm.

Example #3: We are given the rotational inertia, angular displacement, and time, and are looking for the torque from the water. Looks like we need to call back to our rotational kinematic equations! Because we have the displacement and the time, and know that the wheel starts from rest, we can find the angular acceleration. The wheel makes 12 revolutions after 30 seconds, and since a revolution is a full turn of a circle and a circle has 2π radians, we need to multiply 12 by 2π for the angular displacement in radians. This gives us an angular displacement of approximately 75.4 radians.

$$\theta_f = \theta_i + \omega t + \frac{1}{2}\alpha t^2$$

$$(75.4\, rad) = (0\, rad) + (0\, rad/s)(30\, s) + \frac{1}{2}\alpha(30\, s)^2$$

$$(75.4\, rad) = \frac{1}{2}\alpha(900s^2)$$

When we solve for the angular acceleration, we get a value of about 0.17 rad/s². Now that we have the angular acceleration and the rotational inertia, we can simply plug in to Newton's Second Law to find the torque.

$$\tau = I \; x \; \alpha$$

$$\tau = (5000 \; kg \cdot m^2) \;\; x \; (0.17 \; rad/s^2)$$

Our final value for the torque caused by the water is equal to 850 Nm.

5.6

Rotational Motion

Angular Momentum

Angular momentum is exactly what it sounds like: momentum about an axis. Looking at our definition of momentum, if we just substitute our linear values for angular values, we find that angular momentum is:

$$L = I\ x\ \omega$$

Where L is angular momentum, I is rotational inertia, and ω is angular velocity.

Also, using Newton's Second Law for torque, we can also find a definition for the change in angular momentum. With $\Delta L = I\Delta\omega$ and $\tau\Delta t = \Delta L$.

If you notice, the mathematical definitions for angular momentum are similar to linear momentum, just with the values switching. Then, if we set our torque to 0, we see this:

$$0\ \cdot \Delta t = \Delta L$$

$$0 = \Delta L$$

We see that if the net torque on the system is 0, then there will be no change in momentum. In other words, the final angular momentum will be equal to the initial angular momentum should there be no net torque acting upon the system. This is the principle of the

conservation of angular momentum and applies in all cases and all systems in classical mechanics. The units for angular momentum are $kg \cdot m^2/s$. Let's try some practice problems based on this concept!

Example #1: A wheel with rotational inertia 15 $kg \cdot m^2$ spins at a constant angular velocity of 3 rad/s. What is the wheel's angular momentum?

Angular Momentum: ____________ $kg \cdot m^2/s$

Example #2: What is the change in angular momentum in a screw after a person applies a torque of 14 Nm for 4 seconds?

Change in Angular Momentum: __________ $kg \cdot m^2/s$

Answer Key:

Example #1: We can just plug in the given values into our definition for angular momentum.

$$L = I\ x\ \omega$$

$$L = (15\ kg \cdot m^2)\ \ x\ (3\ rad/s)$$

When we solve for the angular momentum of the wheel, we find that it is 45 kg·m²/s.

Example #2: The change in angular momentum is equal to the torque times time, which conveniently happens to be the values we are given in the example. Therefore, all we have to do is plug into the equation.

$\Delta L = \tau \Delta t$

$\Delta L = (14\ Nm)(4\ s)$

When we solve for the change in the angular momentum, we find that the value is equal to 56 kg·m²/s.

5.7

Rotational Motion

Rotational Kinetic Energy

Finally, let's talk about the role energy plays in rotational motion. We know that linear kinetic energy is equal to the value $\frac{1}{2}mv^2$, but the velocity is linear, meaning that we cannot use it while observing rotational motion. We talked about how rotational inertia plays the role of mass in angular motion, so we need to use it instead of mass.

By doing these modifications, we get the following value: $\frac{1}{2}I\omega^2$. This is what we call **rotational kinetic energy**. Rotational kinetic energy is the energy of motion of objects moving about an axis. The change of rotational kinetic energy is also equal to the work done by net torque, just as the change of linear kinetic energy is equal to the work done by net force.

Work done by torque is similar to work done by force. As you might guess, it involves replacing the linear values from our work by force equation with angular values. This results in work by torque being given by the equation $W = \tau\Delta\theta$. The units for this version of energy are still of course Joules (J)! Let's try some practice problems that involve rotational energy!

Example #1: A disk with rotational inertia 2.4 kg·m² spins at a constant angular velocity of 20 rad/s. What is the disk's rotational kinetic energy?

Rotational Kinetic Energy: ________________ J

Example #2: A ball with rotational inertia 5 kg·m² is rolling with an angular speed of 2 rad/s when a person exerts a torque of 30 Nm on it. This torque is applied to the ball for an angular displacement of 22 rads. What is the new angular velocity of the ball?

Angular Velocity: ________________ rad/s

Answer Key:

Example #1: In this example, we're given all the values we need to find the rotational kinetic energy: the rotational inertia and the angular velocity. So, let's do it!

$$KE = \frac{1}{2} I\omega^2$$

$$KE = \frac{1}{2}(2.4\ kg \cdot m^2)(20\ rad/s)^2$$

When we solve for the rotational kinetic energy, we find that the value is equal to 480 J.

Example #2: In this problem, we need to find the final angular velocity, which we can do by finding the final rotational kinetic energy. Let's brainstorm ways to do this. So, in this problem, we're given the rotational inertia and the initial angular velocity. This means we can find the initial rotational KE. However, how can we translate the initial KE to the final KE? In the example, we know the torque applied and the angular displacement of the ball. Earlier, we learned that work done by torque is equal to torque times angular displacement, and work is equal to the change in KE. Thus, we can find the initial KE, add the work done by torque to find the final KE, and from there find the final angular velocity. Let's try it out!

$$KE = \frac{1}{2} I\omega^2$$

$$KE_i = \frac{1}{2}(5\ \mathrm{kg} \cdot m^2)(2\ rad/s)^2$$

The initial kinetic energy is equal to 10 J. Now let's solve for the work!

$$W = \tau \Delta\theta$$

$$W = (30\, Nm)(22\, rad)$$

The value of the work is equal to 660 J, and so the change in kinetic energy is also 660 J. All we need to do now is just add the change in energy to the initial energy to find the final energy, and then the final angular velocity.

$$KE_f = KE_i + \Delta KE$$

$$KE_f = (10\, J) + (660\, J)$$

The value of the final kinetic energy is equal to 670 J.

Now we can solve for the final angular velocity by using our rotational kinetic energy equation.

$$\frac{1}{2} I {\omega_f}^2 = 670\, J$$

$$\frac{1}{2}(5\text{ kg}\cdot m^2)\, {\omega_f}^2 = 670\, J$$

When we solve for the final angular velocity, we see that it is equal to approximately 16.4 rad/s!

5.8

Rotational Motion

Tangential Values

We learned that radians are equal to arc length divided by the radius. This means that we can find out how much distance an object has traveled (the arc length) by knowing how many radians it traveled. Hold on… if we know the distance the object has traveled and the time it took to get there… can't we find the velocity of the object? But how does that work?

Whenever an object spins about an axis, it has something called a **tangential velocity**. This is the velocity of the object tangential to the circular motion of the object and describes the rate of distance traveled.

$$\frac{\Delta\theta}{t} = \frac{\Delta d}{tr}$$

$$\omega = \frac{v}{r}$$

We find that the angular velocity is equal to the tangential velocity divided by the radius. Now, what if we divide by time again to find out what the angular acceleration is equal to?

$$\frac{\Delta\omega}{t} = \frac{\Delta v}{tr}$$

$$\alpha = \frac{a}{r}$$

The angular acceleration is equal to the tangential acceleration divided by the radius. Noticing a pattern? The angular value is equal to the tangential value divided by the radius, and this rule applies to **every value**, so long as they describe the rate of change of the previous one!

Chapter 6

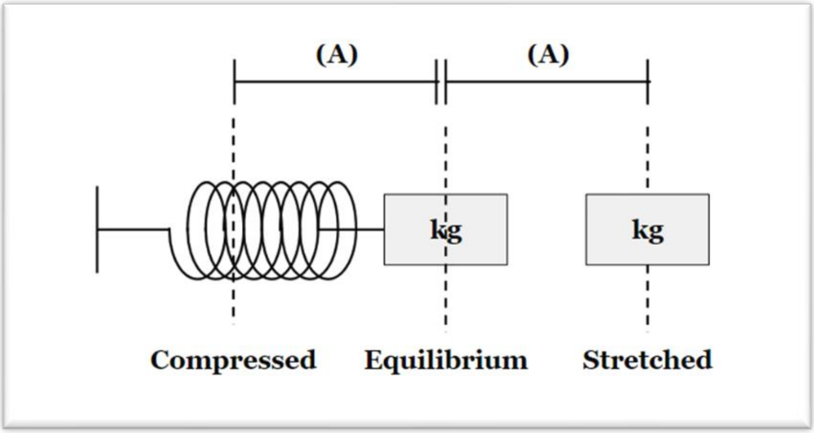
Simple Harmonic Motion

6.1

Simple Harmonic Motion

Introduction to Simple Harmonic Motion

When we talk about Simple Harmonic Motion, SHM for short, we talk about oscillating systems. Oscillating means something that moves or swings back and forth at a constant rate. The two types of systems we will be talking about are mass-spring systems and pendulums. We have already talked about some of these systems in previous units so this unit will be pretty short and straight forward. Before we learn about the various equations behind SHM, we need to learn the basic variables and concepts behind SHM. Let's use a mass-spring system to demonstrate these various ideas. The first idea we will be talking about is Amplitude (A). Amplitude is the maximum displacement from the equilibrium position. For a mass-spring system it is the distance a spring is stretched or compressed from its equilibrium (rest) position. We previously defined this in our Hooke's Law equation, but we called it "x" for displacement, but the concept is the exact same. The units for amplitude would just be meters. Our next variable is Period (T), this essentially is the value for the length it takes to make one complete cycle of motion. For a mass-spring system, it will be the time it takes for the mass to move from its maximum displacement position, back to its equilibrium position, then to its minimum displacement position, back to its equilibrium position, and finally back to its maximum displacement position. The units for the period are seconds. Here's a diagram to help visualize this:



If we know the period (T) of a system, we can find its frequency (f). Frequency is the number of cycles per unit of time which would be seconds. How we find frequency is we find the reciprocal of the period so essentially; we divide 1 over the period ($f = \frac{1}{T}$). But this equation works the other way as well, period is equal to 1 over frequency ($f = \frac{1}{f}$). The units for frequency can either be 1/s or Hz. Let's use a simple example to understand the uses of all these variables and ideas:

Example... A 30 kg block is attached to a spring which stretches a distance of 1 meter. It takes 2 seconds to complete half of a cycle. Find the period (T), frequency (f), and amplitude (A) of the mass-spring system.

To answer this first let's understand what amplitude is, it is the maximum displacement from the equilibrium position. Since the block is stretched 1 meter, we can say that the amplitude is 1 meter. The period is the time it takes for one full cycle to be completed, since it takes 2 seconds for half a cycle to be completed, we can deduce that it would take 4 seconds for a full cycle to be completed so the period would be equal to 4 seconds. Now since we have the period we can solve for frequency, which is how many cycles happen per second, by finding the reciprocal of the period which would be 1/4. This means that the frequency is equal to 0.25 Hz or 1 cycle every 4 seconds. We can also find the period again since we know the frequency by finding the reciprocal of frequency which would be 1/0.25 which is equal to 4 seconds as we expected. The basics of SHM are pretty simple but we can learn how other variables can affect these various parts that make up SHM!

6.2

Simple Harmonic Motion

The Mass-Spring System

Boing... Boing... Boing... That's the sound of a mass-spring system oscillating from side to side. The main aspect to learn about in the mass-spring system is how the period of a mass-spring system is affected by different variables. Let's think, what could change in a mass and a spring for its period of oscillation to change? Well of course we can assume the mass change will affect the period and also the spring constant of the spring. Well, the equation for the period of a mass-spring system is almost just that! The actual equation is equal to $T = 2\pi\sqrt{\frac{m}{k}}$. Since we have the equation for the period, the frequency as we know is just the reciprocal of this value. There is no need to memorize the frequency equation as we can simply just find the reciprocal of the period after using the period equation. Again, just to recap our period equation lets us know the time it takes for the mass-spring system to complete one full oscillation from the maximum displacement position back to the maximum displacement position after going through the equilibrium and minimum displacement positions. Let's take a look at an example:

Example... A 1 kg mass is on a frictionless surface attached to a spring, compressed leftward and released. If the spring constant is equal to 200 N/m and it has a maximum compression length of 0.1 meters, determine the period and frequency of oscillation of the mass.

To solve this question let's utilize our new equation to solve for the period and later for the frequency. Once we plug in the known variables, we get an equation that looks like this: $T = 2\pi\sqrt{\frac{(1\,kg)}{(200\,N/m)}}$, this gives us a period of approximately 0.44 seconds. To find the frequency all we have to do is divide 1 by our period, 1/0.44 gives us a frequency of about 2.25 Hz. As we will see after doing more SHM questions, this unit is pretty straightforward and utilizes just a few basic concepts and equations. Let's take a look at some examples of the mass-spring system…

Example #1: Liana stretches a spring with a 2 kg mass attached to it on a frictionless table. If the spring has a spring constant of 100 N/m, find the period and frequency of oscillation of the mass-spring system.

Period: _____________ Seconds

Frequency: ____________ Hz

Example #2: Kareem has a 500 N/m spring that oscillates with a Frequency of 2 Hz which has an unknown mass attached to the end, he is currently looking for the mass of the unknown mass, help Kareem find the value of unknown mass.

Mass of the Unknown Mass: ___________ Kg

Answer Key:

Question #1: To find the period and frequency of this mass-spring system we can simply use our period equation to find the period and later find the reciprocal of that number to find the frequency. Let's plug in our knowns into our period equation, it will look something like this:

$T = 2\pi\sqrt{\frac{(2\ kg)}{(100\frac{N}{m})}}$we get a period of about 0.89 seconds. When we find the reciprocal of this value by dividing 1 by 0.89, we get a value of about 1.13 Hz which would be the frequency.

Question #2: This question is a little bit trickier than the previous question as now the frequency is given to us, and we are solving for one of the unknown variables. Let's first solve for the period by finding the reciprocal of 2 Hz which would be 1/2 which is 0.5 seconds. Now that we have that, let's plug in the parts of the equation which are known so we can isolate the equation for the unknown which is the value of the unknown mass. The equation will look like this: $0.5\ s = 2\pi\sqrt{\frac{m}{(500\ N/m)}}$, now this question has just turned into a basic algebra question. Let's isolate the equation for the value of "m". First, divide both sides by 2π, and we get $0.0796 = \sqrt{\frac{m}{(500\ N/m)}}$. After solving for the value of m, we get a value for the unknown mass which is equal to about 3.17 kg!

6.3

Simple Harmonic Motion

The Simple Pendulum

Have you ever been on a swing at a park? A swing is essentially a large pendulum. In this topic, we will be learning about the physics of a pendulum. The period of a pendulum depends on many things, let's think about what could affect its period. First, the length of the pendulum would definitely affect how long it takes to swing from its maximum displacement position back to its original position. Another thing that can affect it is the force that accelerates the pendulum, this force is of course the force of gravity. If we combine all these various ideas, we can get an equation for the period of a pendulum which is equal to $T = 2\pi\sqrt{\frac{L}{g}}$. L in the equation is equal to the length of the pendulum arm. Of course, the frequency is the reciprocal of this equation just like in the mass-spring system period equation, so we do not need to memorize it! Let's take a look at a basic example of how we can use this new equation.

Example... Don wants to find the period of a simple pendulum with an arm length of 2 meters on Mars. The acceleration due to gravity on Mars is equal to 3.72 m/s^2. Find the period of the pendulum and the frequency.

Let's use our new equation to find the period first and then the frequency of this pendulum. Let's plug in our known, $T = 2\pi\sqrt{\frac{(2\ m)}{(3.72\ m/s^2)}}$, if we solve for the period we get a value of 4.61 seconds. If we want to find the frequency, we will simply just divide 1 by 4.61 to get a value of 0.22 Hz.

Now let's say that Don travels to Earth and wants to see the new period of the pendulum.

Let's solve this equation by using 10 m/s^2 as our gravity instead of 3.72 m/s^2. $T = 2\pi\sqrt{\frac{(2\ m)}{(10\ m/s^2)}}$, we get a value of 2.81 seconds. This means that if we are on a planet with a stronger acceleration due to gravity, the period of a pendulum will be less than if we were on a planet with a weaker acceleration due to gravity. Let's try some more questions:

Example #1: Brandon hops on a spaceship and heads for a distant planet. He wonders which planet he is on once he lands. Luckily, he has his trusty pendulum and his stopwatch. He knows the length of his pendulum is 0.192 meters and when he measures the period he gets 3.5 seconds. Which planet is he on?

A) Mars - 3.72 m/s^2

B) Earth - 9.81 m/s^2

C) Pluto - 0.62 m/s^2

D) Jupiter - 25.7 m/s^2

Example #2: Joe wants to find the length of his pendulum on Jupiter which has an acceleration due to gravity of 25.7 m/s^2, he measures the frequency of his pendulum as 0.25 Hz. Find the length of the pendulum.

Length of the Pendulum: _______________ m

Answer Key:

Question #1: To solve this question, we have to consider the parts of the equation which we already have, we have the length of the pendulum and the period. We are looking for the gravity so let's set up our equation by keeping in mind that we are solving for the gravity or "g" in the equation. $3.5\,s = 2\pi\sqrt{\frac{(0.192\,m)}{g}}$, now to solve for gravity we will first divide both sides of the equation by 2π, when we do that we get an equation that looks like this: 0.557 = $\sqrt{\frac{(0.192\,m)}{g}}$, to get rid of the square root we will square both sides of the equation, we will get $0.31 = \frac{(0.192\,m)}{g}$. Now we can multiply both sides 0.31 by g to remove g from the left side, we will get $0.31g = 0.192$. Finally, if we divide both sides by 0.31, we get a value of g which is about 0.62 m/s^2 which is equal to the acceleration due to gravity on Pluto or answer choice C.

Question #2: To answer this question, we first have to solve for the period of the pendulum. We can do this by simply finding the reciprocal of 0.25 Hz which is equal to $\frac{1}{0.25}$ which is equal to 4. Now that we have the period, let's set up the equation with the variables we have. $4\,s = 2\pi\sqrt{\frac{L}{(25.4\,m/s^2)}}$, since we are solving for the length, we have to isolate the equation for the value of "L". When we solve for the length, we find that the pendulum will have a length of 25.7 meters.

6.4

Simple Harmonic Motion

Energy in Simple Harmonic Motion

When an object is oscillating, it has velocity. This means that energy must be present in some form in oscillating systems, because kinetic energy equals $\frac{1}{2}mv^2$. The types of energy which are present depend on the object which is oscillating.

In mass-spring systems, there is conversion between kinetic energy and elastic energy. Imagine a horizontal spring, with one side attached to a block and the other side attached to a wall. If you pull the block, the elastic potential energy in the block-spring system will increase. Once you release the block, the block-spring system reaches its maximum potential energy. Then, this potential energy gets converted into kinetic energy. When the spring reaches its equilibrium point, there is no more potential energy, thus all of it is kinetic. This means that at the spring's equilibrium point, the system has the greatest velocity. But because the block has inertia, it needs to keep going, compressing the spring causing that kinetic energy to turn back into elastic energy—repeating this cycle.

Meanwhile, in a pendulum, the potential energy is gravitational rather than elastic. At the highest point of the pendulum, it has the greatest potential energy. As the pendulum swings to its equilibrium point, the potential energy is converted to kinetic energy. At the equilibrium point, the kinetic energy is at its maximum as there is no potential energy (the equilibrium point for a pendulum is when it is at its lowest position). When the pendulum swings back up, the kinetic energy is converted to potential and repeats this process.

A Final Note

Well, this is the end of the book. We both hope you enjoyed the way we taught these topics and learned something new. This book took us a long time to write and edit but we both know that it serves as a catalyst to help others learn physics and gain an understanding of it. We noticed many students throughout our years of learning physics in school struggling with it yet enjoying it at the very same time. We wanted to create a method of teaching that is much more relatable to students and sparks that interest in students while removing the difficulty of the subject. Thanks for reading and learning using our book! We hope you can spread our message and support us in our future endeavors. Thank you and continue exploring the mysteries of the universe!

-Rafi and Zenel

About the Author

Rafi Uddin is a student currently at Brooklyn Technical High School in the Aerospace Engineering Major. He lives in New York City, and specifically in Queens. He plans on pursuing a degree in physics in the future in college and hopes to continue teaching students. He is also the current president of the Physics Olympiad team at Brooklyn Tech, find out more about him at https://www.bthsphysicsolympiad.com !

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